جامعة تكريت
كلية الهندسة
تاسست سنة ١٩٨٨

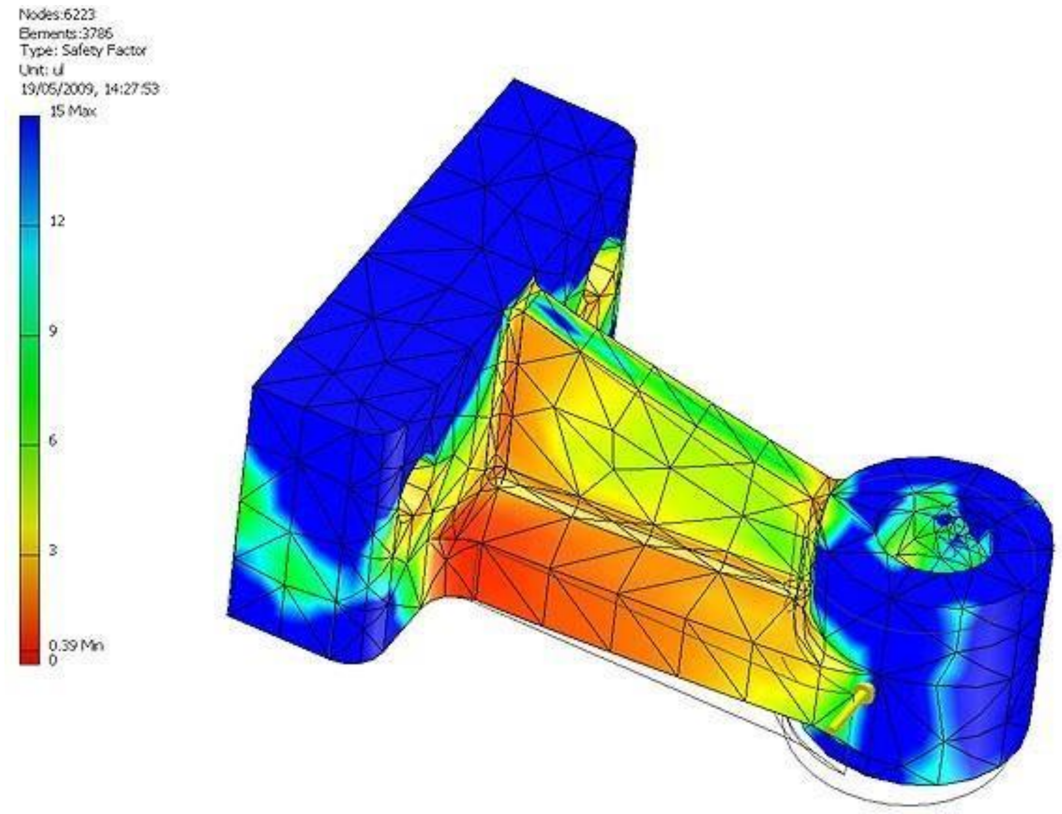
Strength of Materials

Chapter 1

Stress

Engineering mechanics:

- Statics
- Dynamics
- Mechancis of Materials

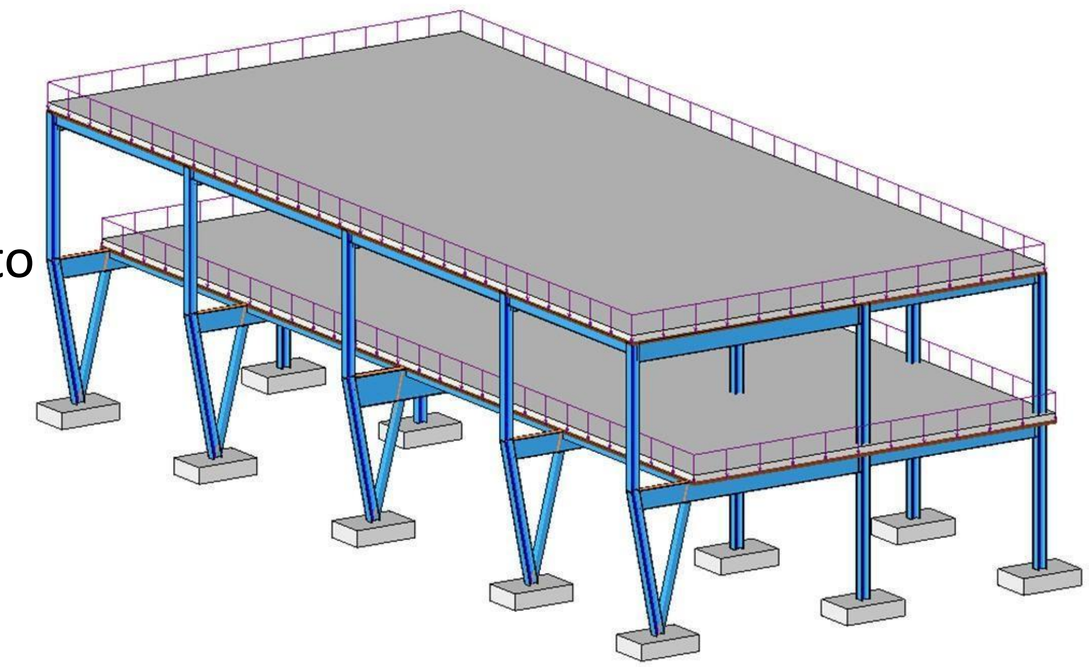


Statics and dynamics: are devoted primarily to the study of external forces and motions associated with particles and **rigid bodies** (i.e., idealized objects in which any change of size or shape due to forces can be neglected)**.**

Mechanics of Materials: is the study of the internal effects caused by external loads acting on real bodies that deform (meaning objects that can stretch, bend, or twist).

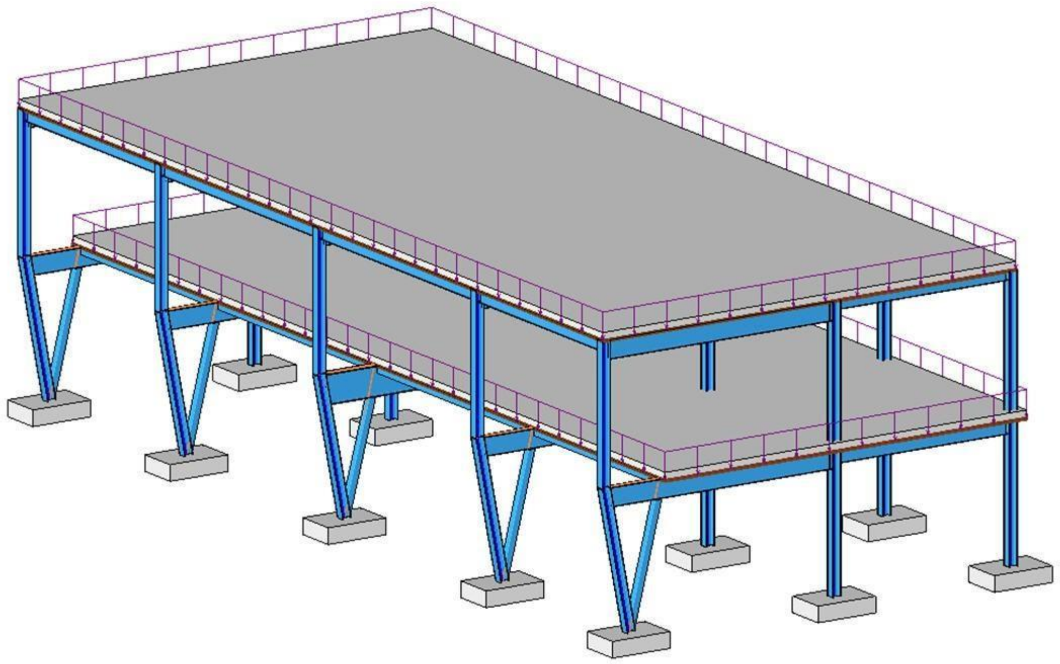
Why are the internal effects in an object important?

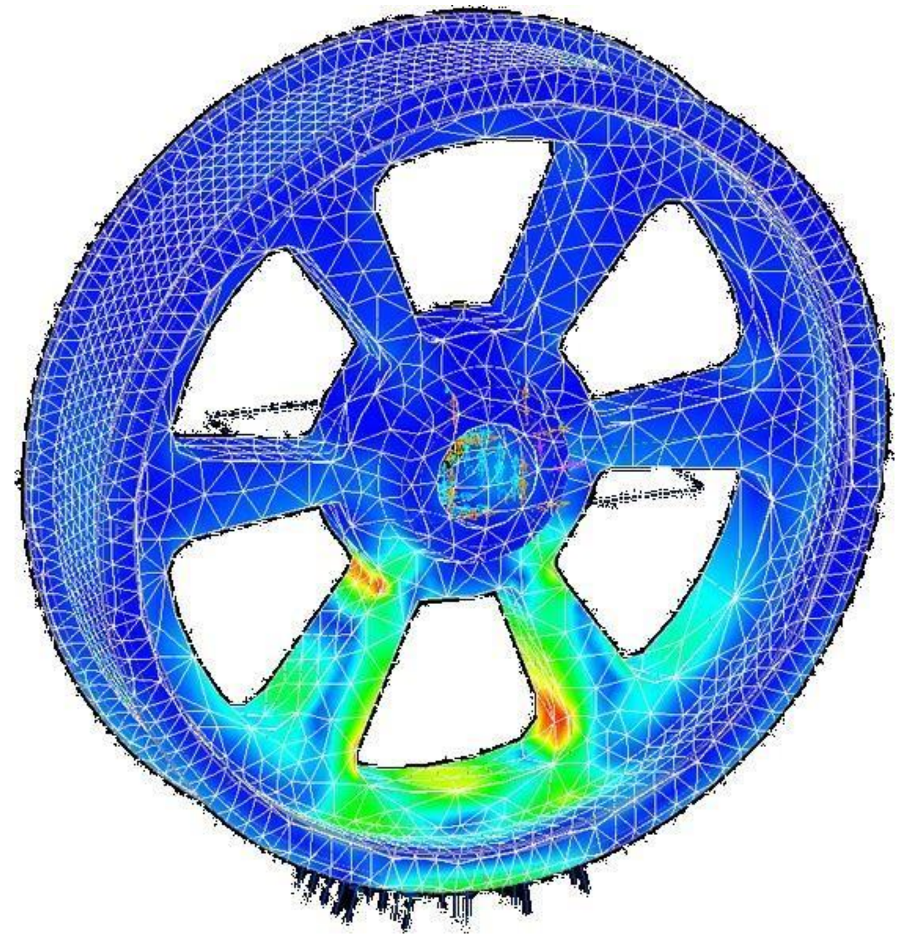
Engineers are called upon to design and produce a variety of objects and structures such as automobiles, airplanes, ships, pipelines, bridges, buildings, tunnels, retaining walls, motors, and machines. Regardless of the application, however, a safe and successful design must address the following three mechanicalconcerns:

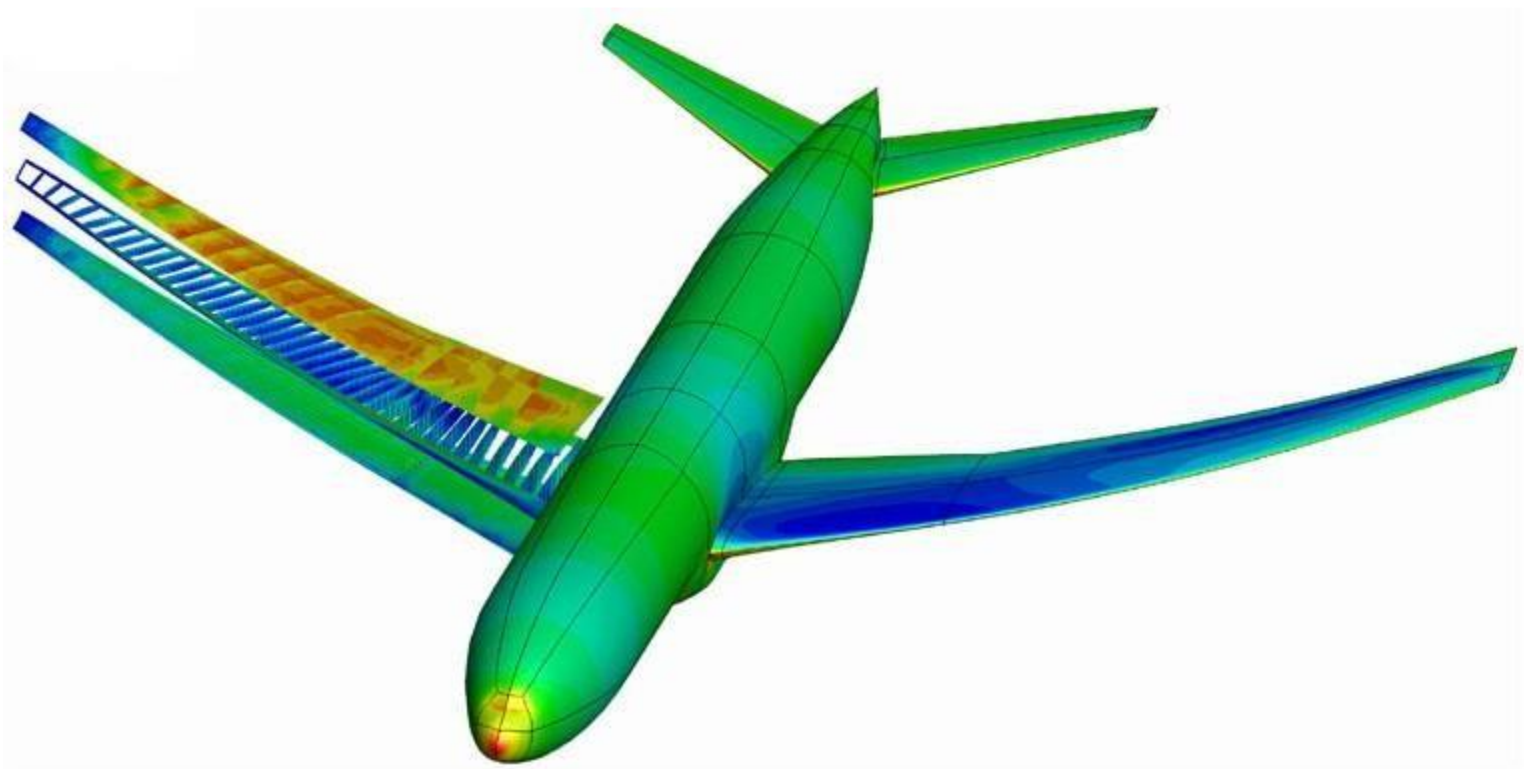


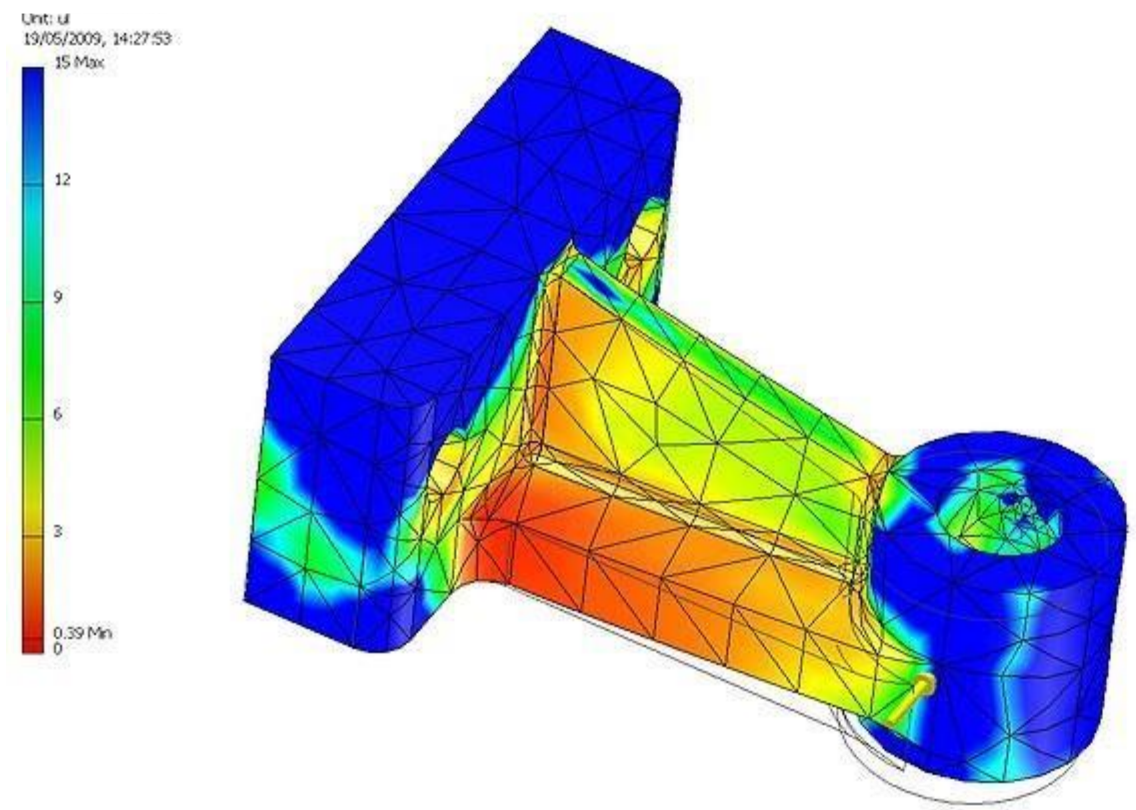
1. Strength: Is the object strong enough to withstand the loads that will be applied to it? Will it break or fracture? Will it continue to perform properly under repeated loadings?
2. Stiffness: Will the object deflect or deform so much that it cannot perform its intended function?
3. Stability: Will the object suddenly bend or buckle out of shape at some elevated load so that it can no longer continue to perform its function?

Mechanics of Materials (Strength of materials) is a fundamental subject which deals with the behavior of solid elements subjected to loads. Distribution of internal forces and deformation caused by such loading is of primary interest. The study of strength of materials often refers to various methods of calculating the **stresses** and **strains** in structural members, such as beams, columns, and shafts.

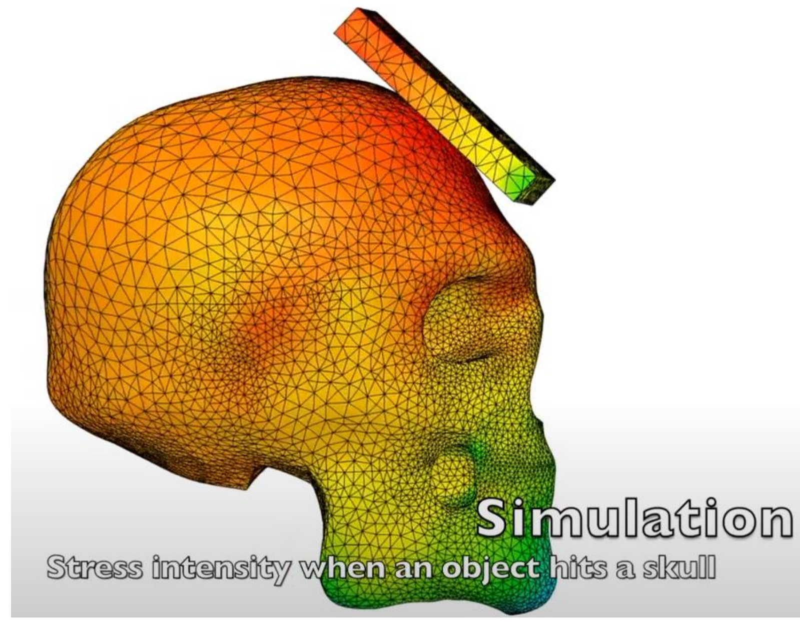


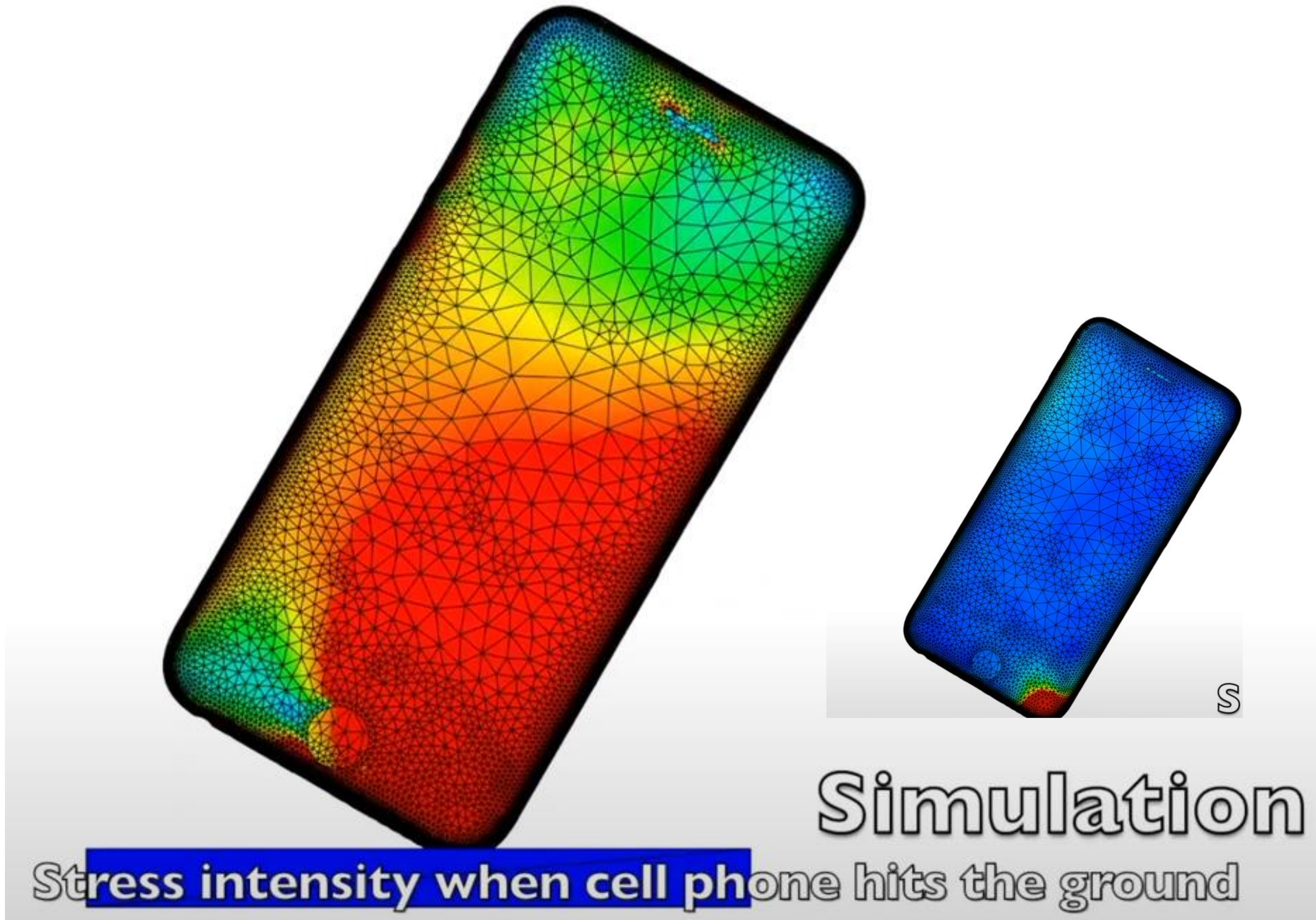






Mechanics of Materials is a fundamental subject which deals with the behavior of solid elements subjected to loads. Distribution of internal forces and deformation caused by such loading is of primary interest.
The study of strength of materials often refers to various methods of calculating the **stresses** and **strains** in structural members, such as beams, columns, and shafts.





Basic elements of **Strength of Materials** are **Stress** and **Strain**.

Stress: intensity of internal force

Strain: intensity of internal deformation

What weighs more? Steel or feather

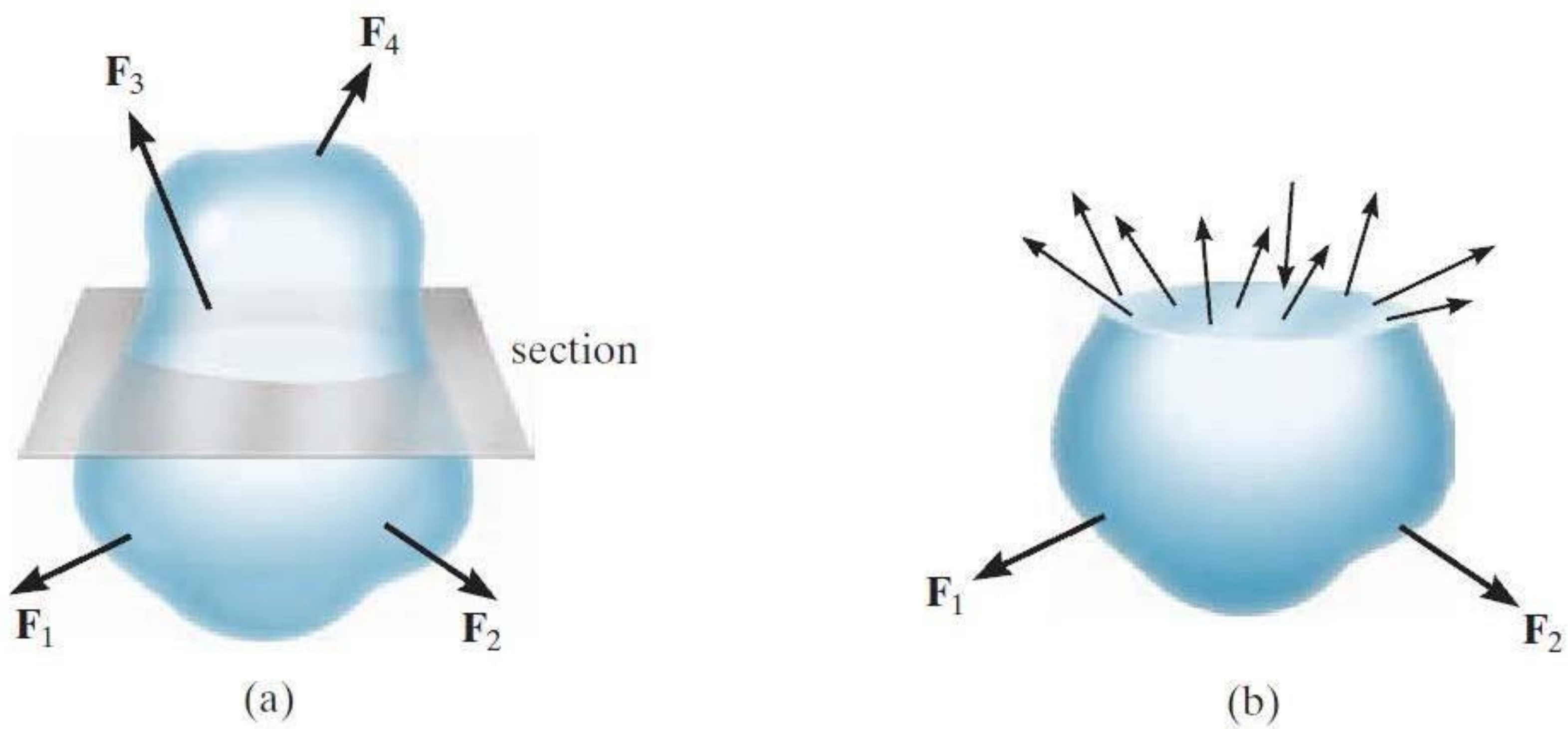
The **density**, or more precisely, the **volumetric mass density**, of a substance is its mass per unit volume.

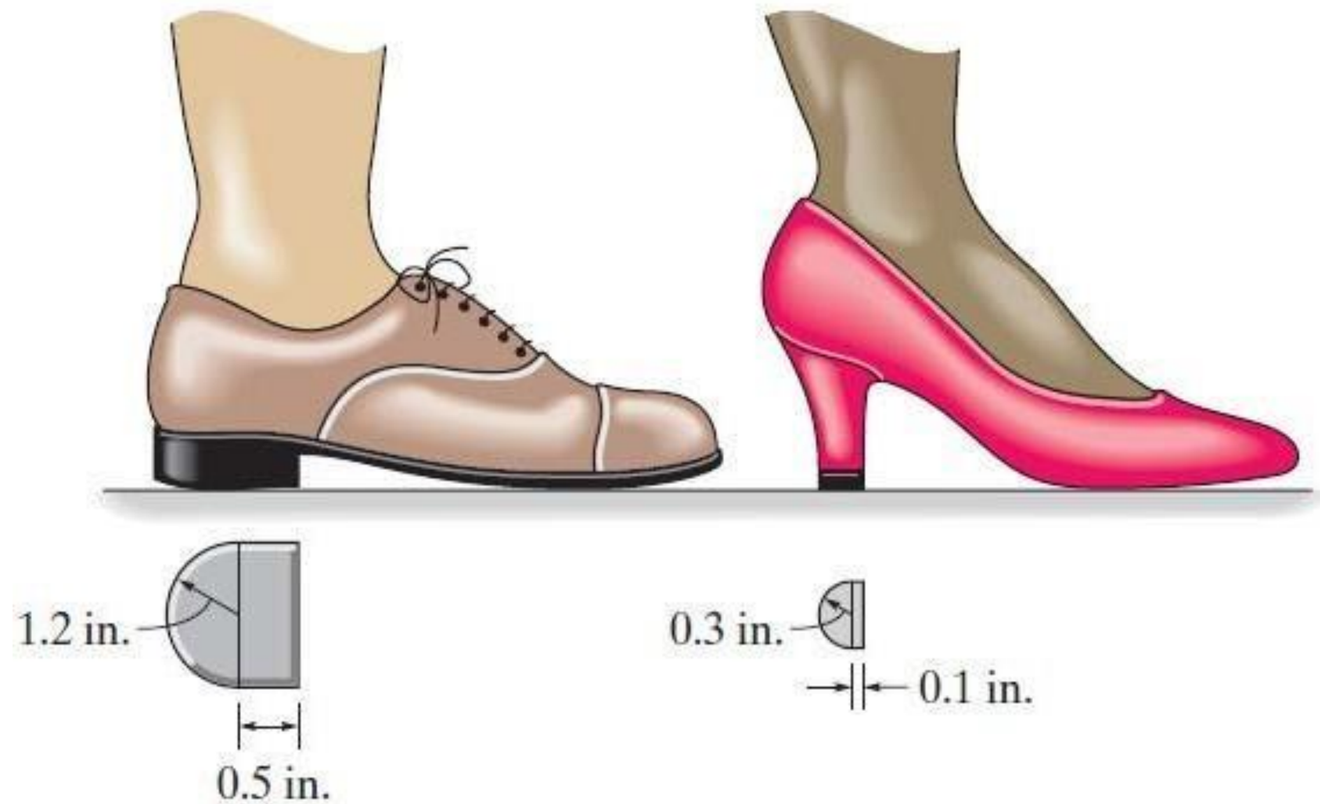
$$\rho = \frac{m}{V}$$

m : mass
V : volume

Which one has higher density?

Which one can resist more load?





$$stress = \frac{Force}{Area}$$

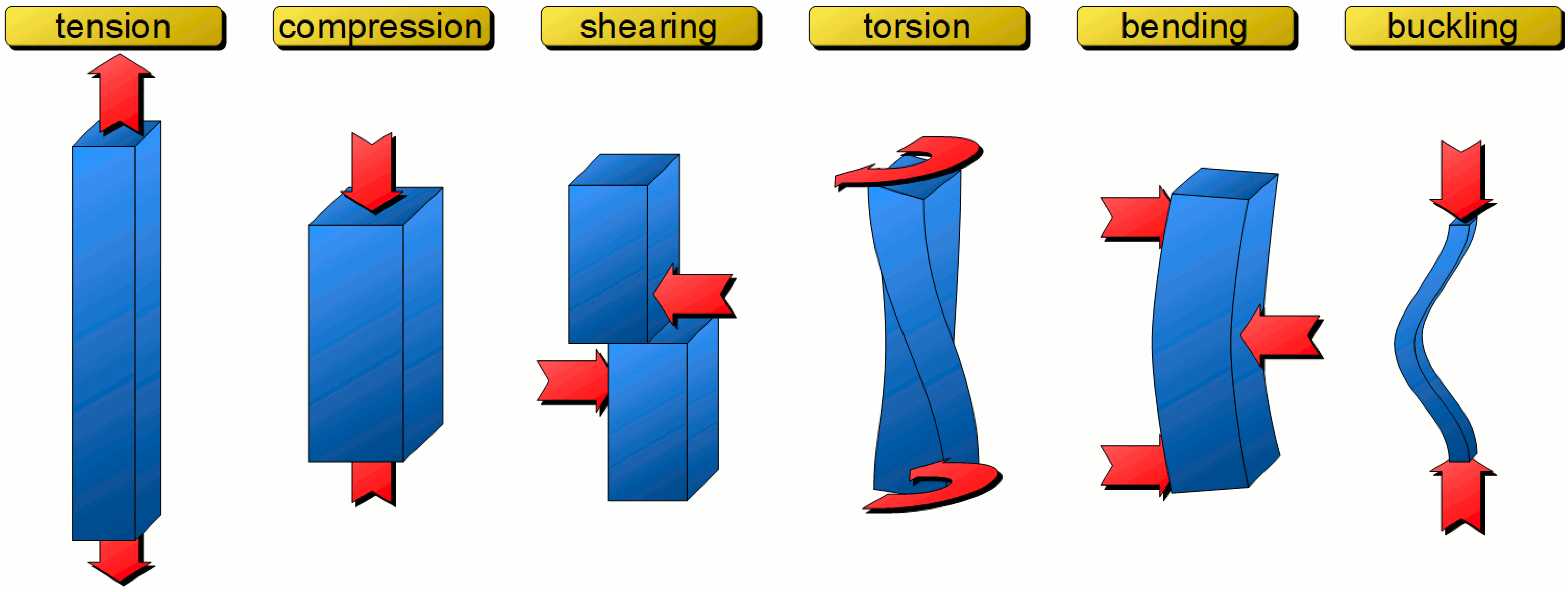
What would be more painful?

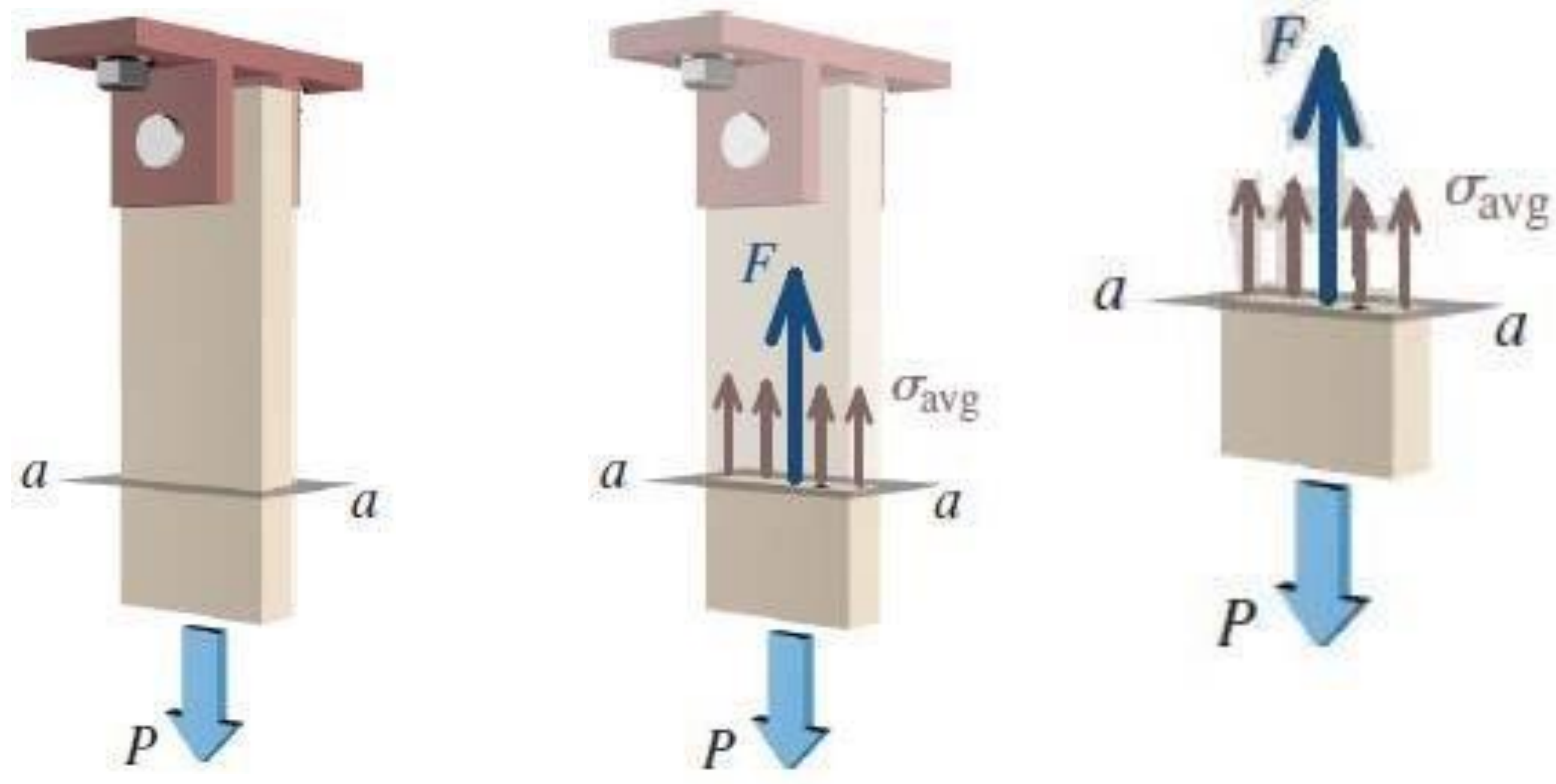
Being trodden on by a 55kg woman wearing stiletto heels?

Or being trodden on by a 3 tonne elephant?

(55*9.81/2)/0.0001 m^2
=2,697,750 N/m^2

(3000*9.81/4)/0.1 m^2
=73,575 N/m^2


tension
compression
shearing
torsion
bending
buckling



Normal Stress

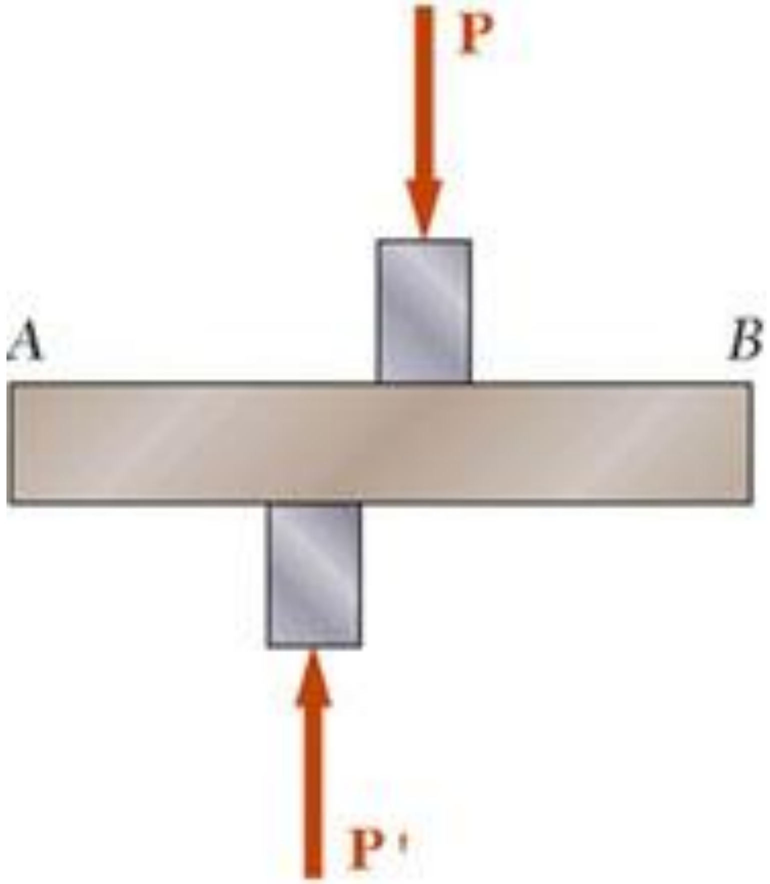
Stress: intensity of internal force

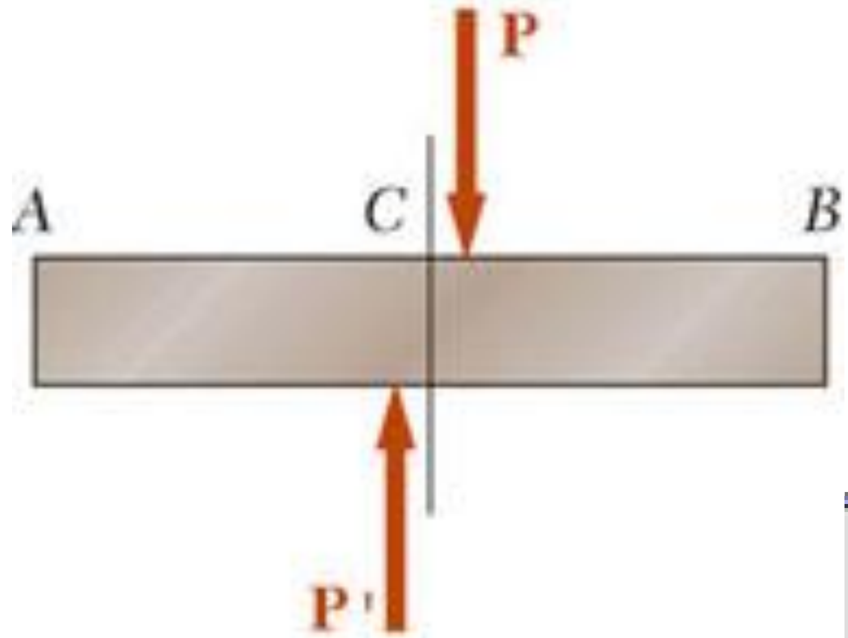
$$stress = \frac{Force}{Area} \qquad \sigma = \frac{F}{A}$$

+ Tension
- Compression

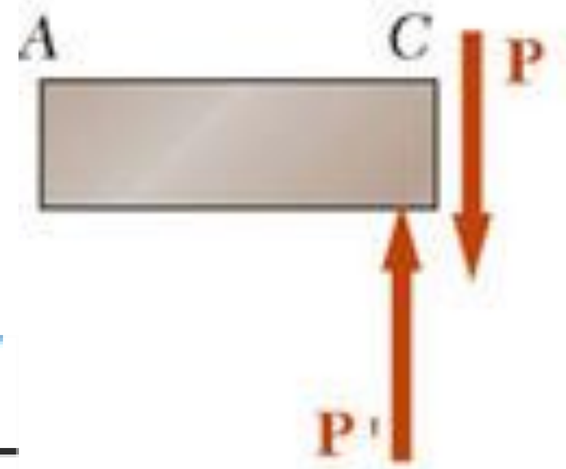
The resultant of the internal forces for an axially loaded member is *normal* to a section cut perpendicular to the member axis.

Shear Stress

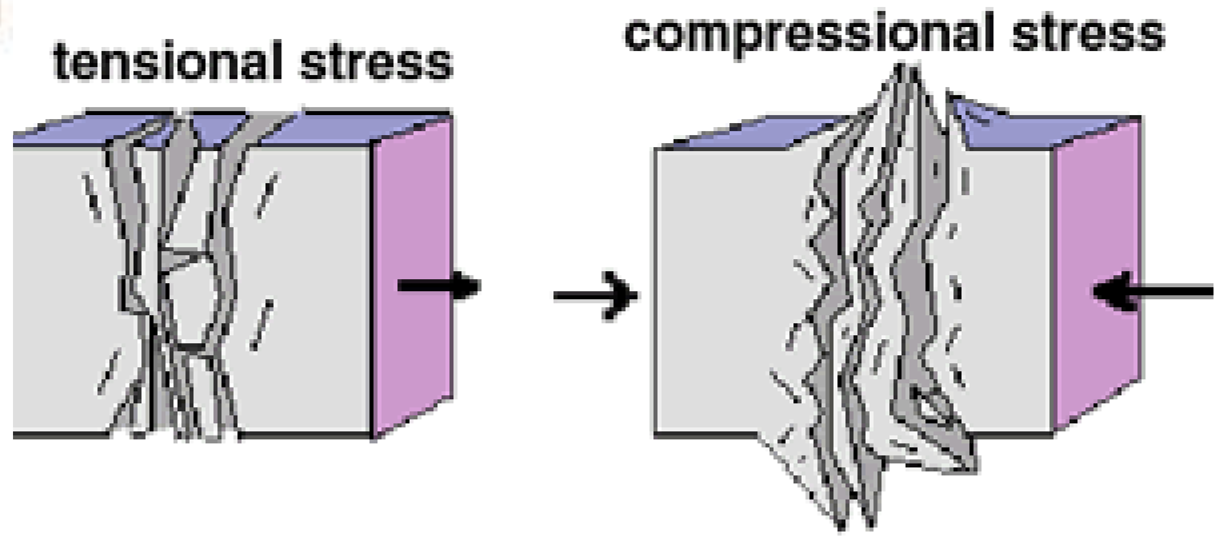


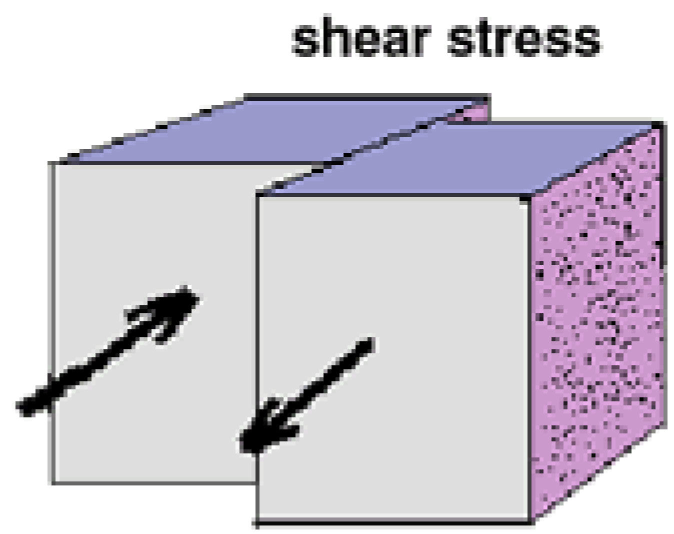


(a)



(b)

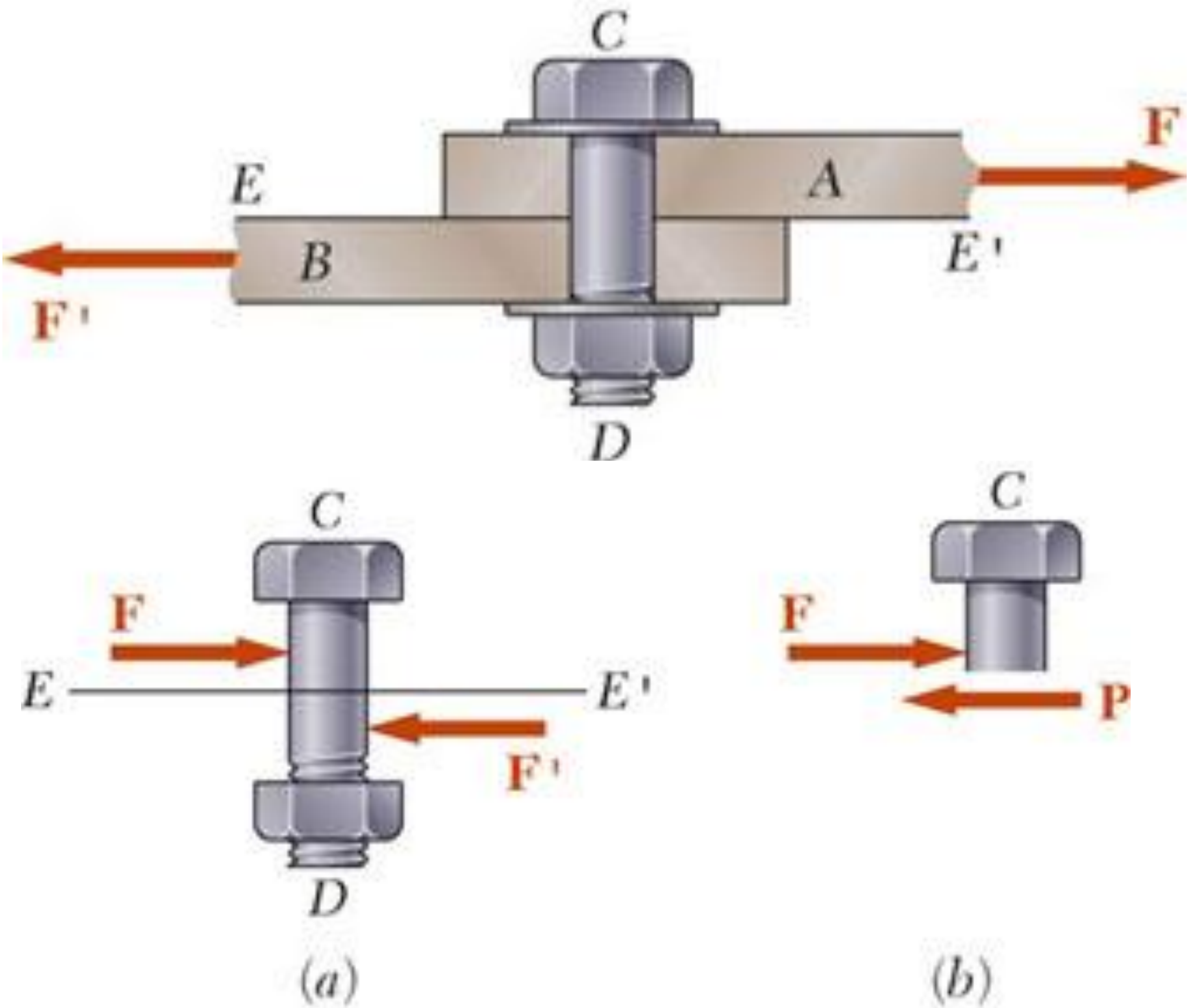




$$stress = \frac{Force}{Area} \qquad \tau = \frac{F}{A}$$

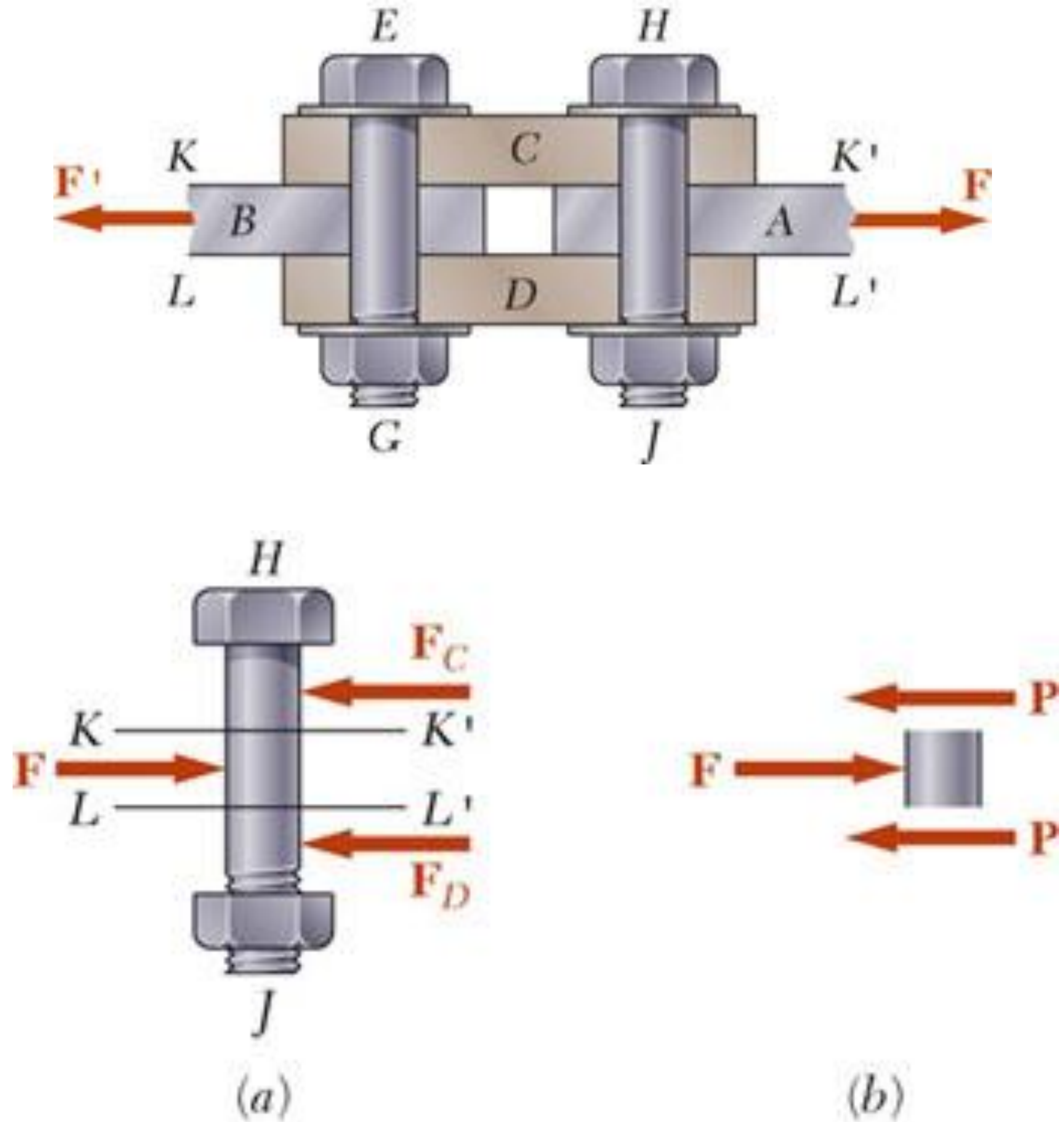
The resultant of the internal forces for loaded member is <u>parallel</u> to a section cut perpendicular to the member axis.

Single Shear



$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F}{A}$$

Double Shear



$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F}{2A}$$

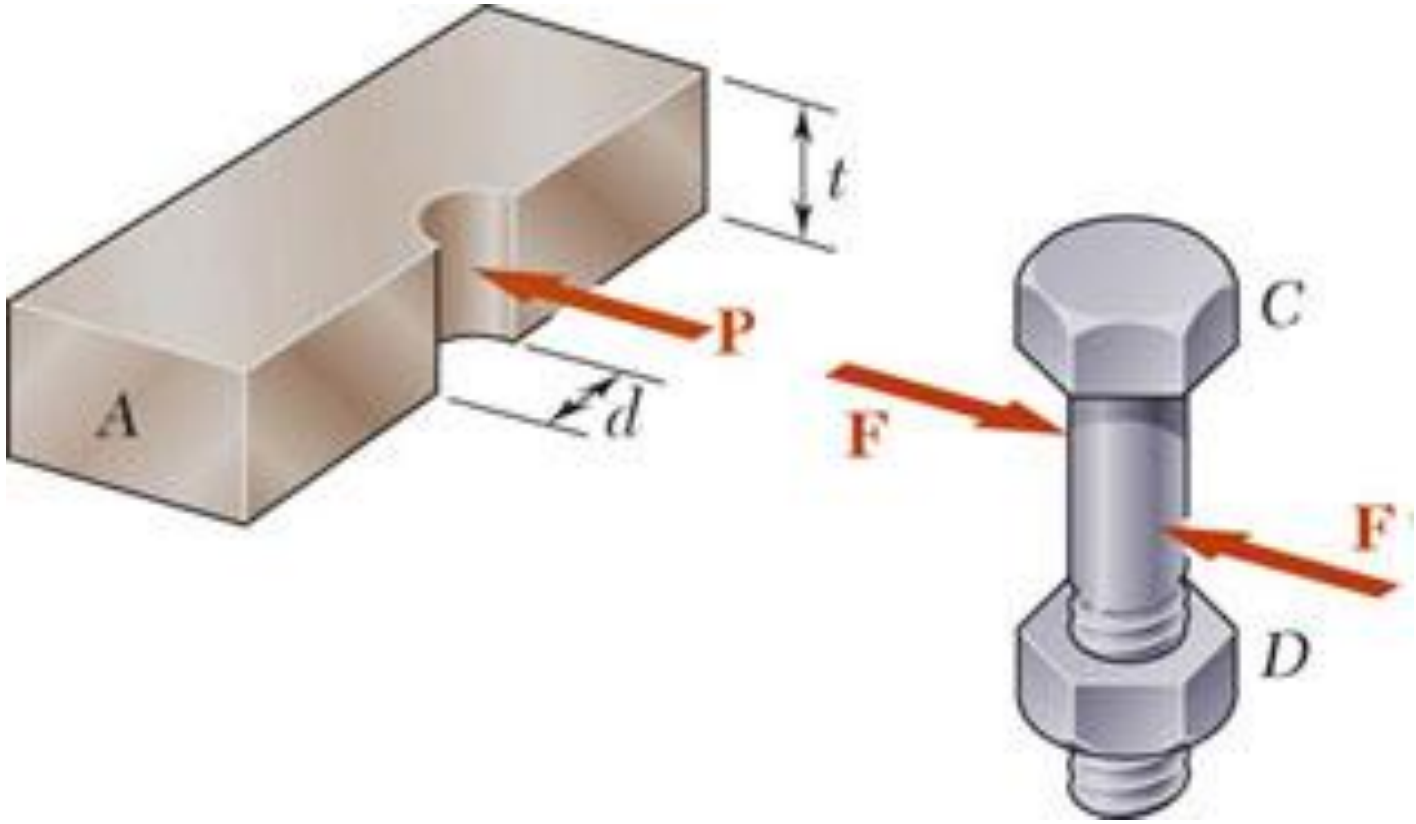
Single Shear

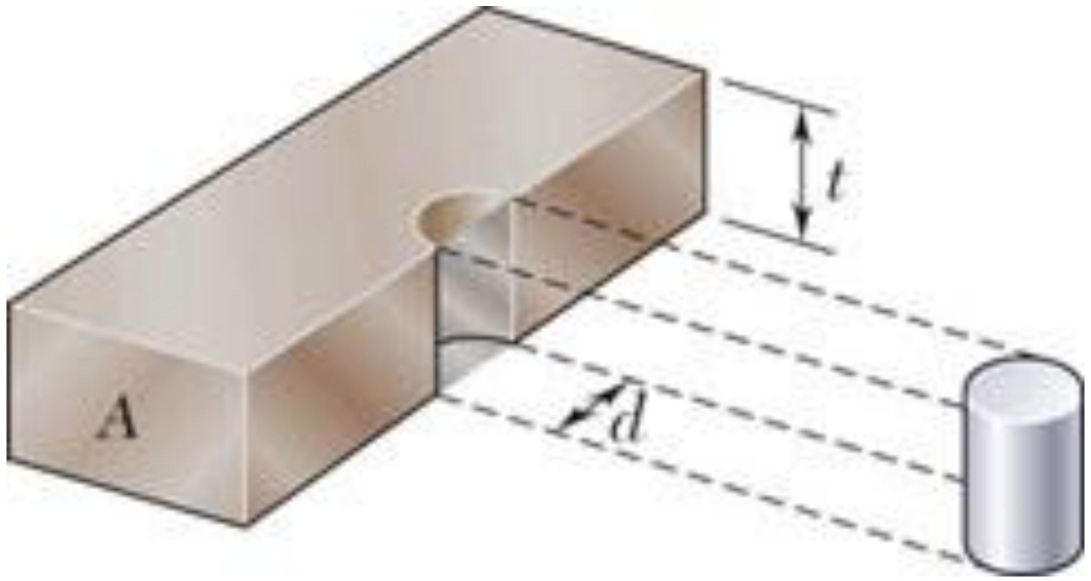
$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F}{A}$$

Double Shear

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F}{2A}$$

Bearing Stress

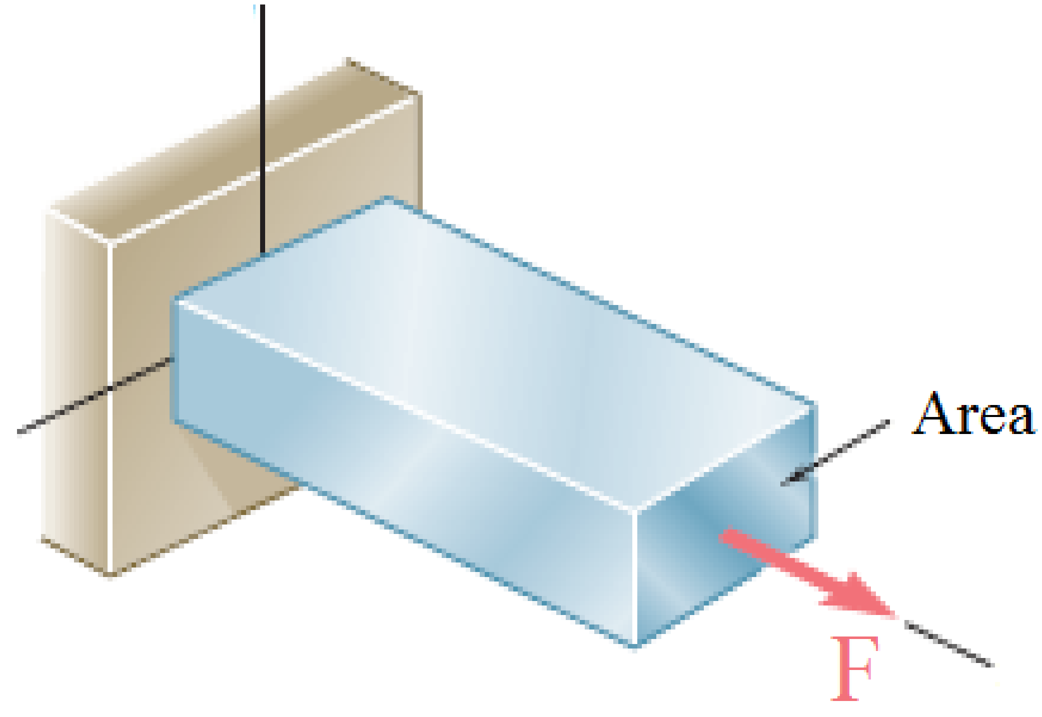




$$\sigma_b = \frac{P}{A} = \frac{P}{t\,d}$$

Bolts, rivets, and pins create stresses on the points of contact or ***bearing surfaces*** of the members they connect.

Stress analysis problems



Step 1) Determine internal force, F (use FBD)

Use Free Body Diagram (FBD) to determine internal force (F) at the point where the stress should be calculated.
See next page for more details on FBD technique.

Step 2) Determine cross section area, A

Determine the cross section area at which the internal force is acting on. Note that the cross section area is usually perpendicular to the axis of elements.

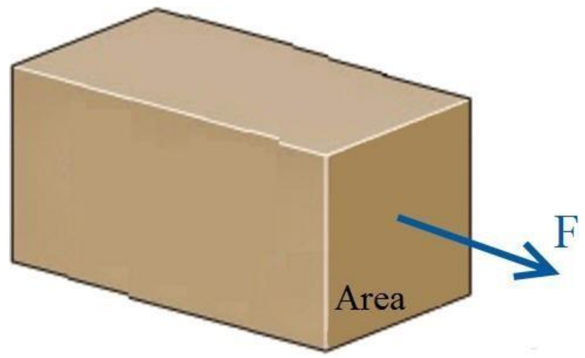
Step 3) Determine stress, σ or τ

Stress is force (F) divided by cross section area (A):

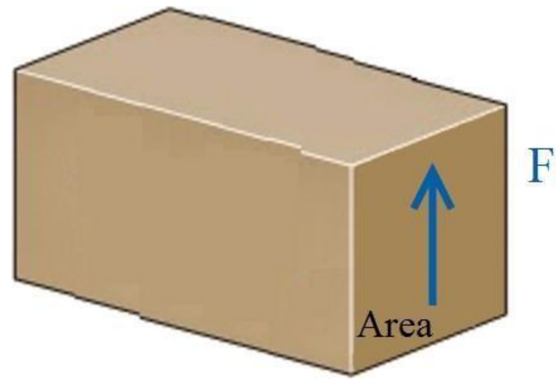
ormal stress: $$\sigma = \frac{F}{A}$$

Shear stress: $$\tau = \frac{F}{A}$$

Note that the equation used for calculating normal stress is similar to the equation used for shear stress.
In normal stress, the force is perpendicular to the area; but in the shear stress the force is parallel to the cross section area.



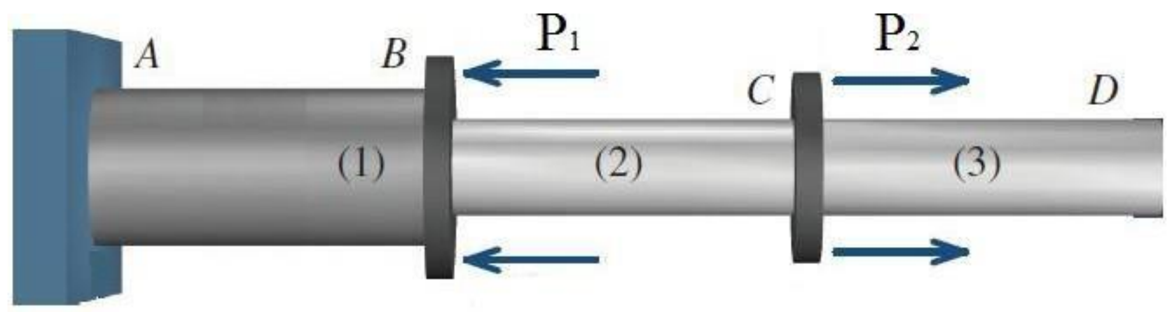
Normal stress:
the force is perpendicular to the area;



Shear stress:
the force is parrllel to the area;

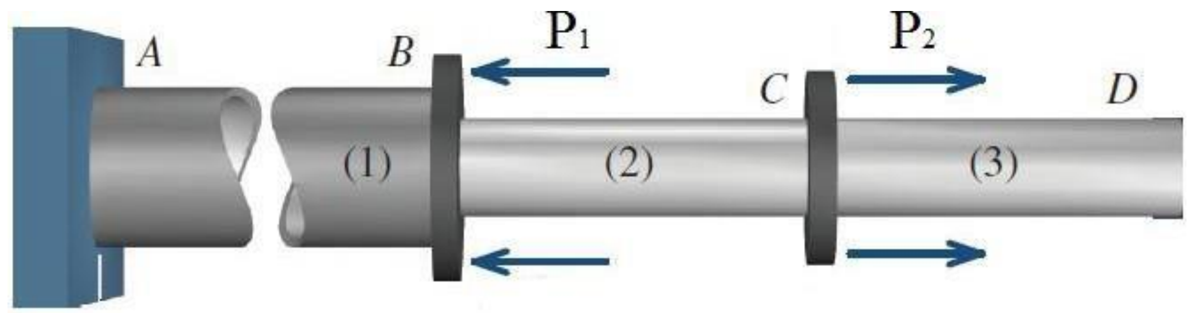
Free Body diagram for determining internal force

This technique is basically used for determining internal force in a determinate structure. The structure is usually restrained by few restraints. The structure is determinate if the number of equilibrium equations are equal to the number of restraints. In 1D axial elements, structures with one restraints are determinate.



Step 1) Cut the element to make it free

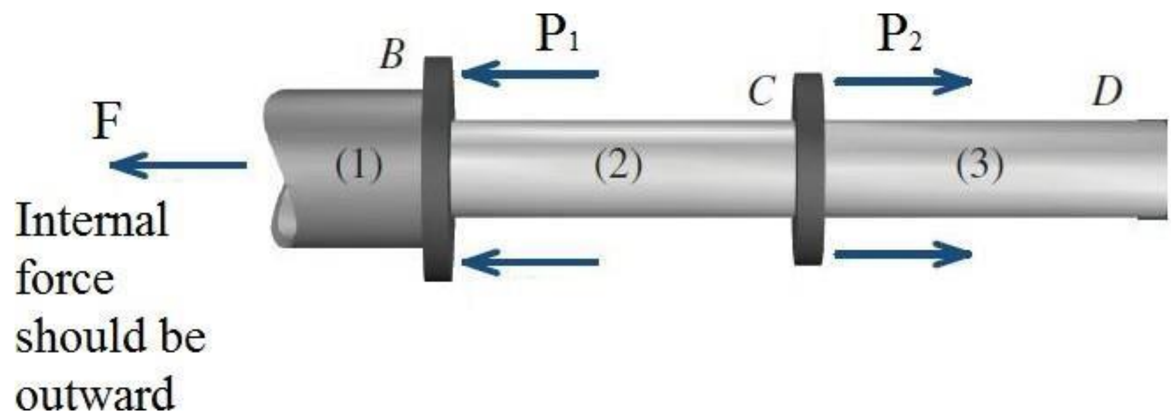
Cut the section at the point at which the internal force shuold be determined.



Step 2) Apply forces

Apply all external forces acting on the cut element. Put an unknown force at the cut section facing outward from the cut section.

Important note:
Consider the free part of structure; the part without restraints



Step 3) Use equilibrium equations

Use equilibrium equation to deterimine the internal force (F).

1-dimensional axial element: $\sum F_x = 0$

2-dimensional structures: $\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M = 0$

Note: After solving equilibrium equation, the sign of resulted internal force (F) shows:

Positive internal force <--> Tension force
Negative internal force <--> Compressive force

Stress is intensity of internal force inside the structure.

Note 1: Sign of stress + Tension
- Compression

Note 2: Appropriate unit for stress

SI unit: MPa

US customary unit: psi or ksi

Note 3: Unit conversion

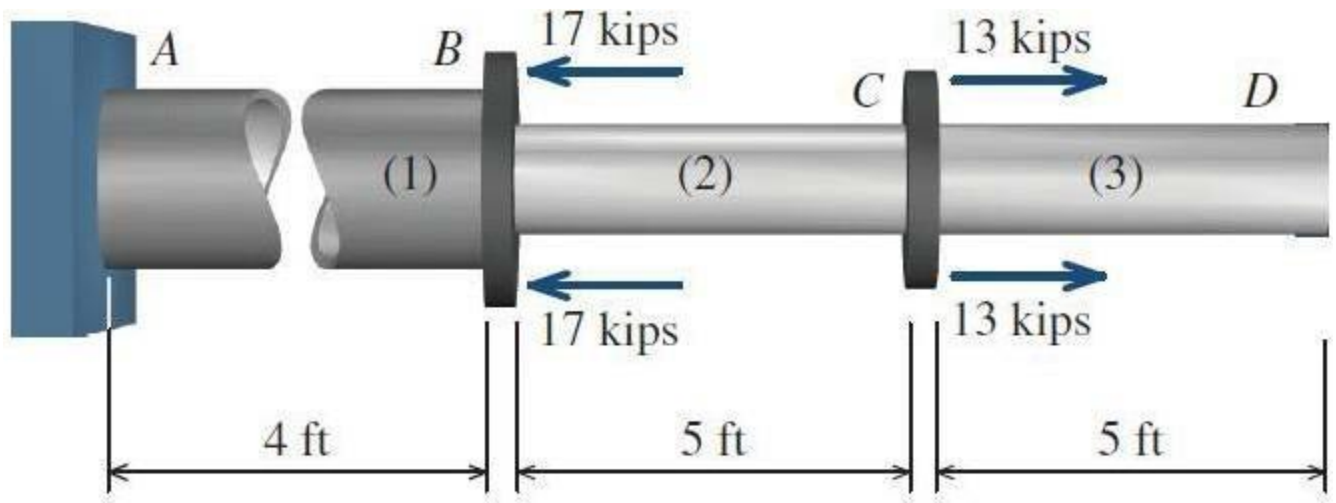
MPa = N/mm^2

1 Mpa = 1,000,000 Pa

psi = lb/in^2

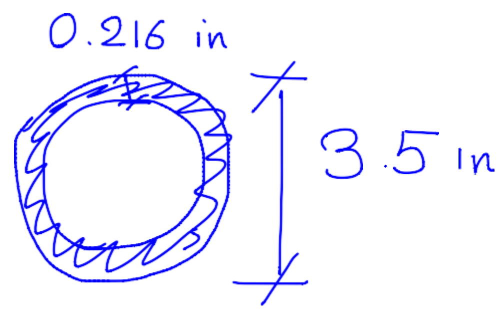
1 ksi = 1,000 psi

Example 1: A hollow steel tube (1) with an outside diameter of 3.50 in. and a wall thickness of 0.216 in. is connected to a solid 2-in.-diameter aluminum rod. The assembly is attached to supports at the left ends and is loaded as shown in the Figure. Right end is free to move. What is the normal stress in element (1).



$$\sigma = \frac{F}{A}$$

Area of element ① (Big circle – small circle)



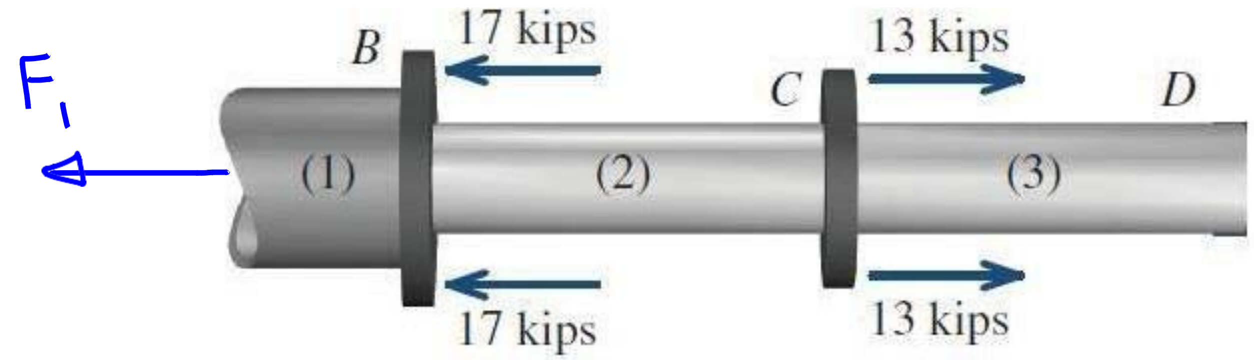
$$A = \frac{\pi}{4}\left[D^2 - d^2\right]$$

$D = 3.5$ in

$d = D - 2t = 3.5 - 2 \times 0.216 = 3.068$ in

$$A = \frac{\pi}{4}\left[3.5^2 - 3.068^2\right] = 2.228 \text{ in}^2$$

force in element ①



$$-F_1 - 17 \times 2 + 13 \times 2 = 0$$

$F_1 = -8$ kips Compression

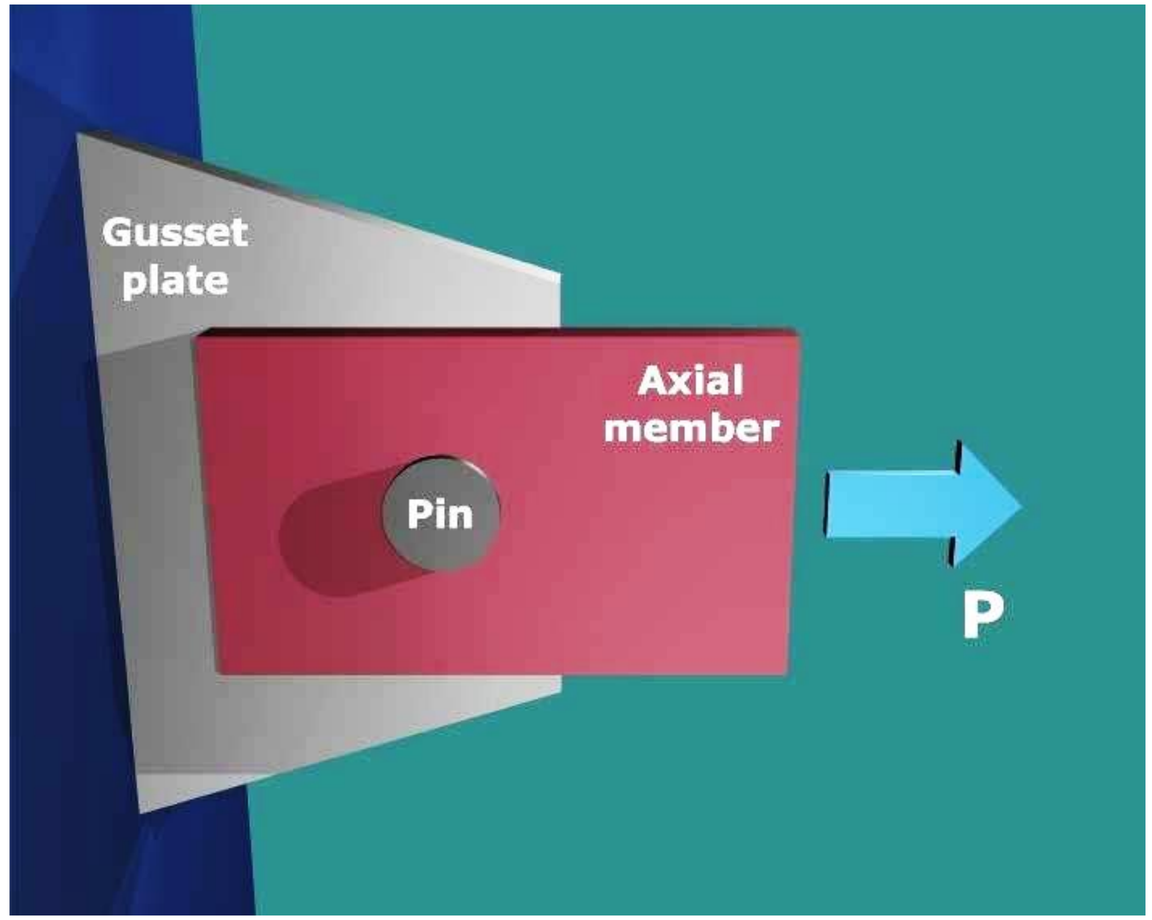
Stress in element ①

1 ksi = 1000 psi

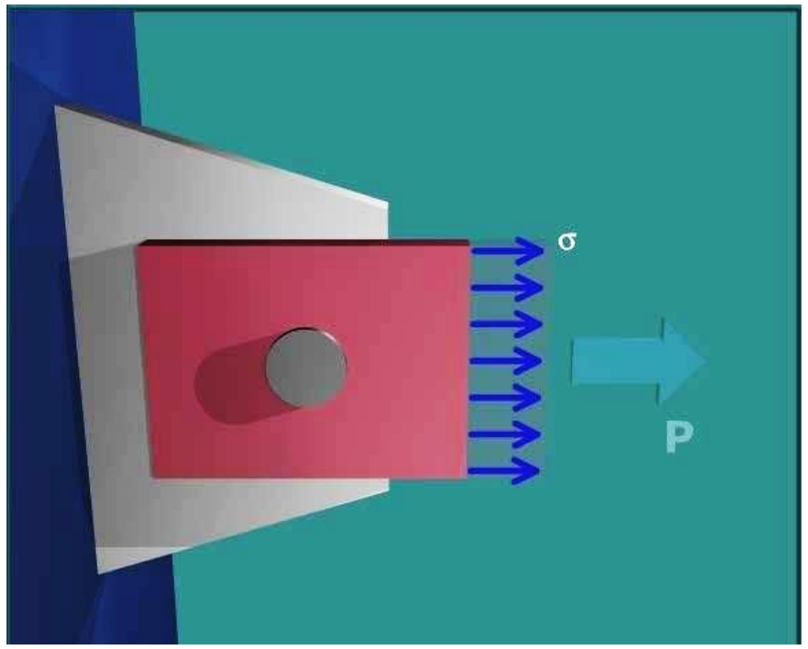
$$\sigma = \frac{F_1}{A_1} = \frac{-8 \text{ kips}}{2.228 \text{ in}^2} = -3.59 \text{ kips/in}^2$$

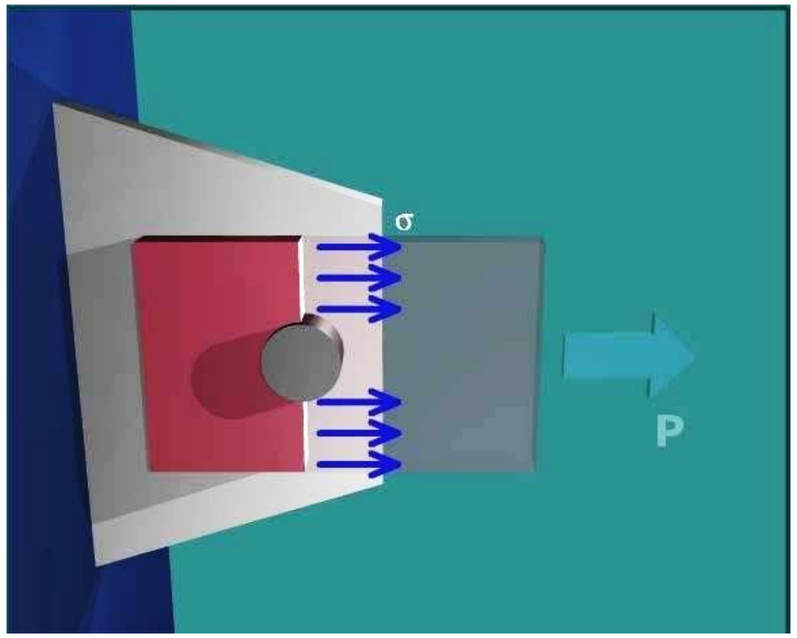
$$= -3590 \text{ psi}$$

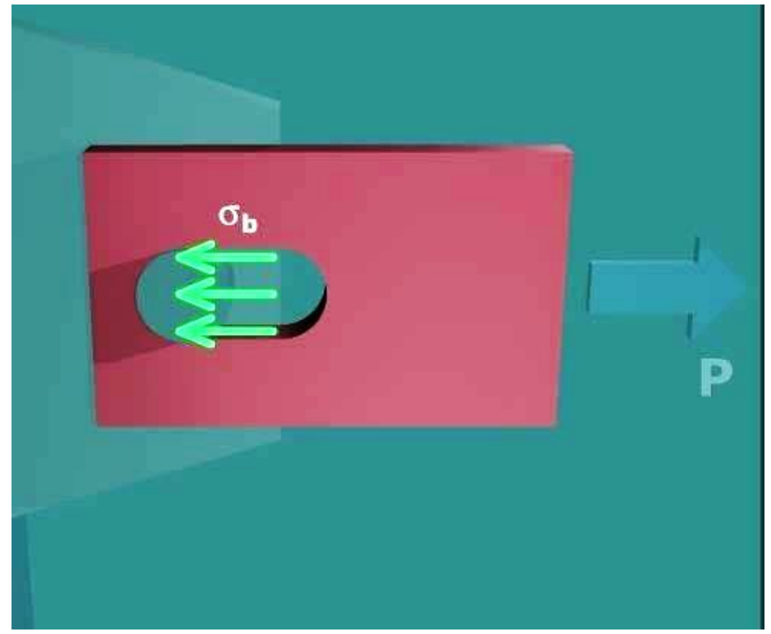
Example 2: Determine the maximum normal stress and shear stress and bearing stress in the connection shown in the figure. Use the following parameters.
P=120 kips.
Axial member: thickness=1/2”, width=10”
Pin: diameter =3/4”



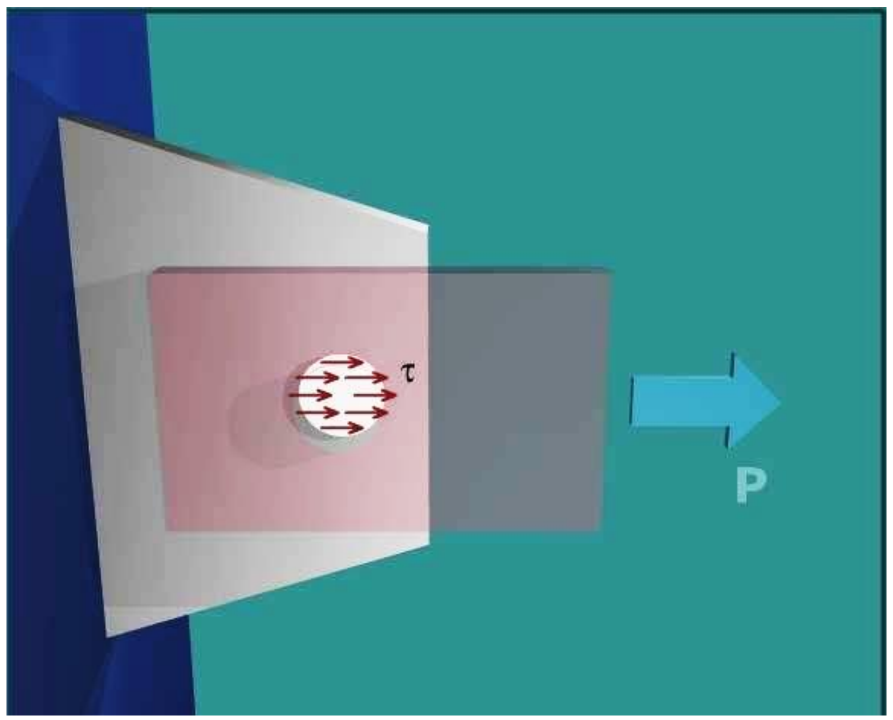
normal stress

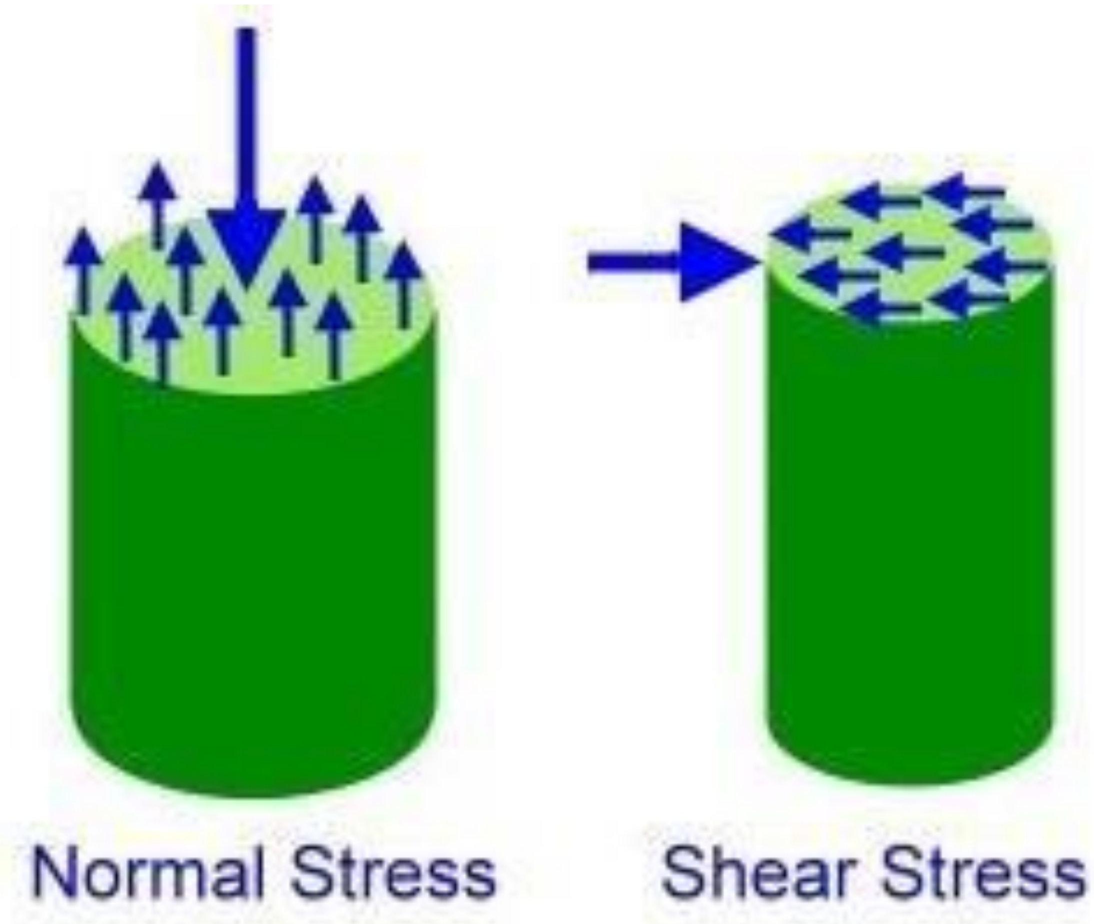




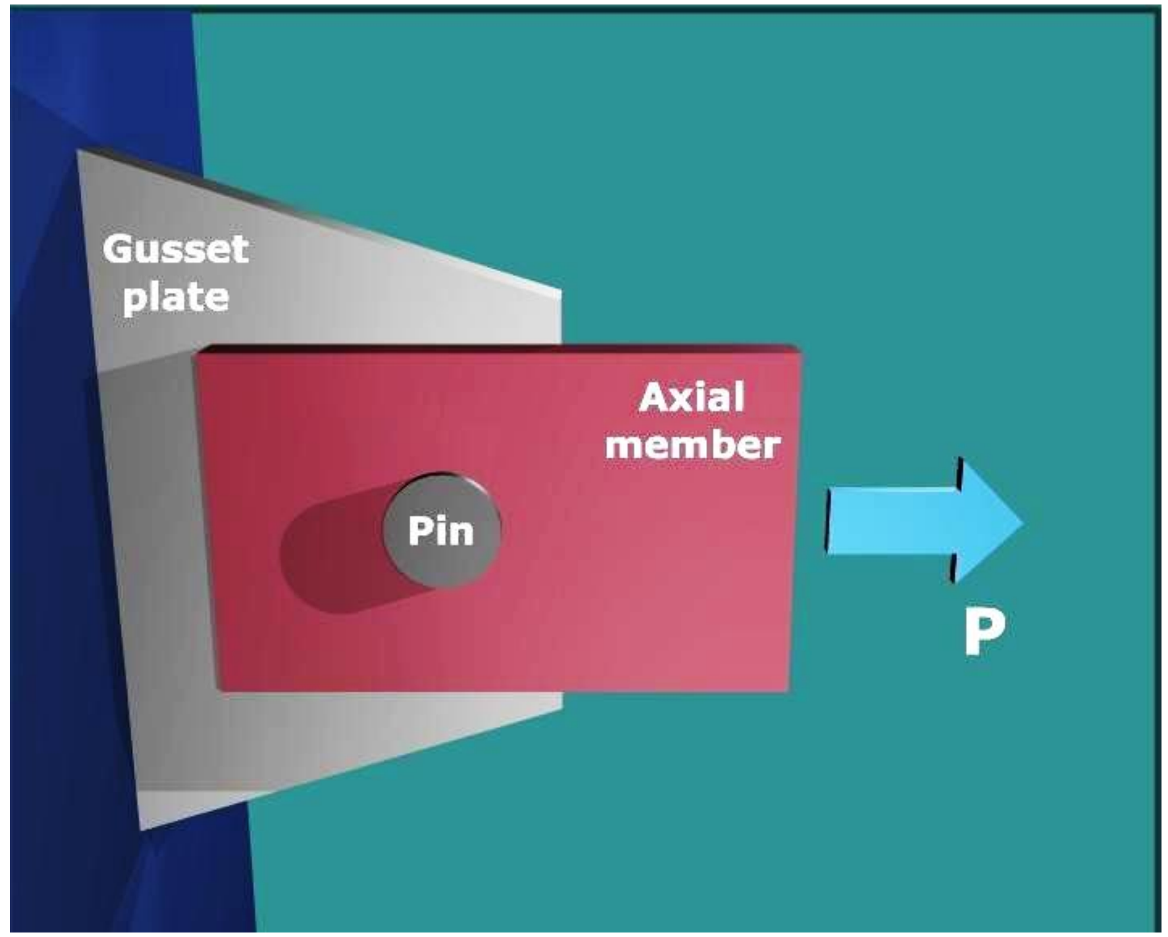


shear stress





Determine the maximum normal stress and shear stress and bearing stress in the connection shown in the figure.
Use the following parameters.
P= 120 kips.
Axial member: thickness=1/2", width=10"
Pin: diameter=3/4"



Part a) Normal Stress

Step (1) Internal force

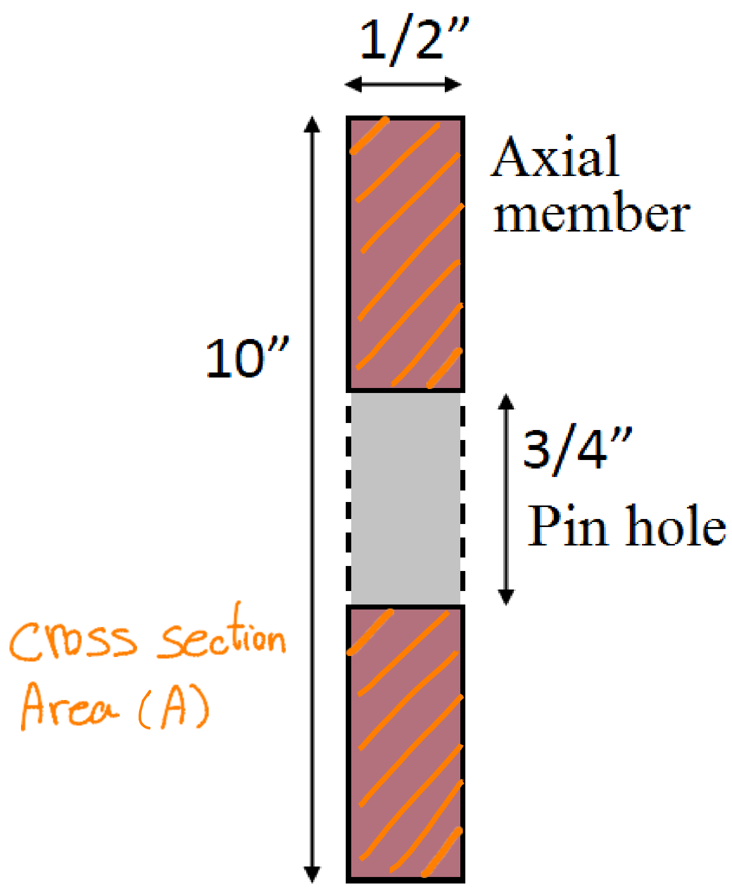
$F = P = 120 \text{ kips} = 120{,}000 \text{ lb}$

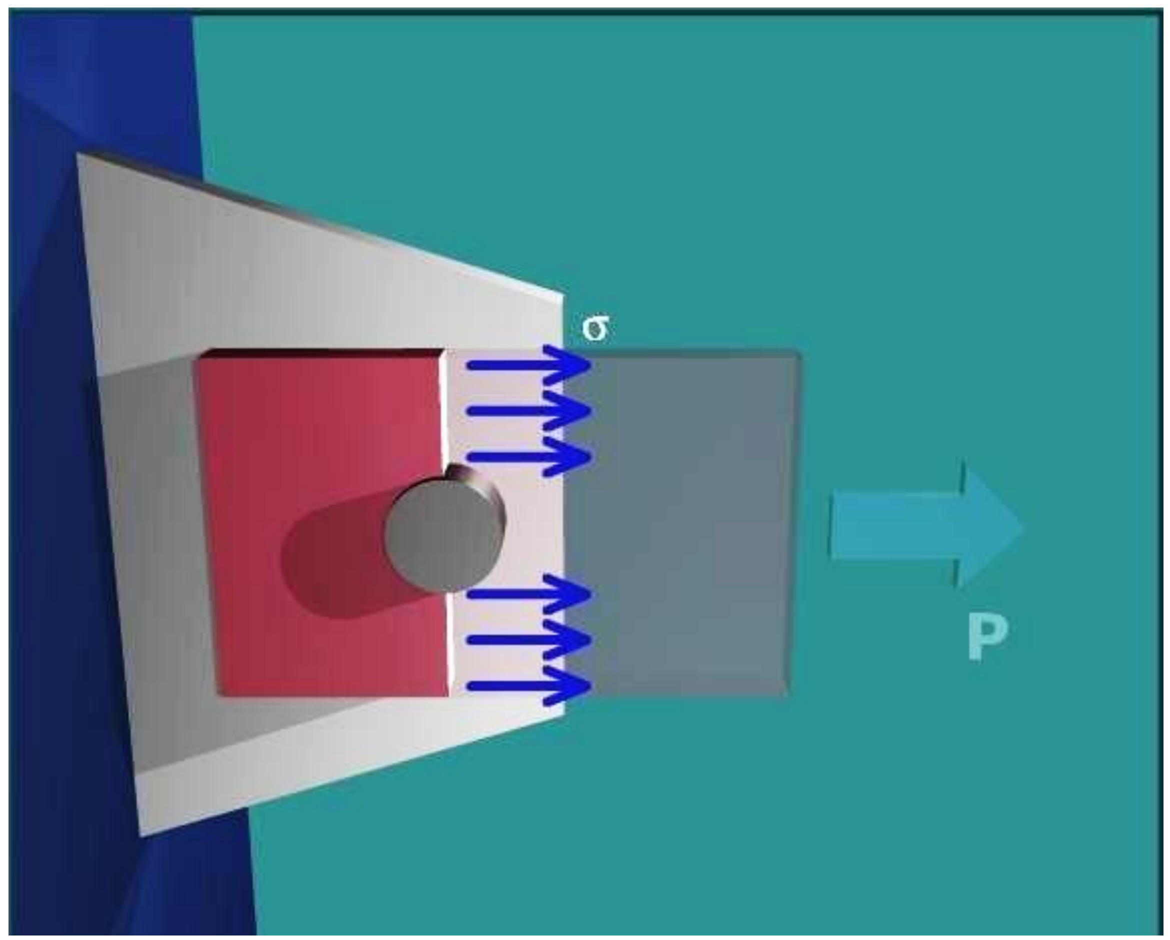
Step (2) Cross section area

A = thickness x effective width

$A = 1/2'' \times (10 - 3/4'')$

$A = 4.625 \text{ in}^2$





Step (3) Normal stress

$\sigma = \frac{F}{A} = \frac{120{,}000 \text{ lb}}{4.625 \text{ in}^2} = 25{,}950 \text{ lb/in}^2$

$\boxed{\sigma = 25{,}950 \text{ psi}}$

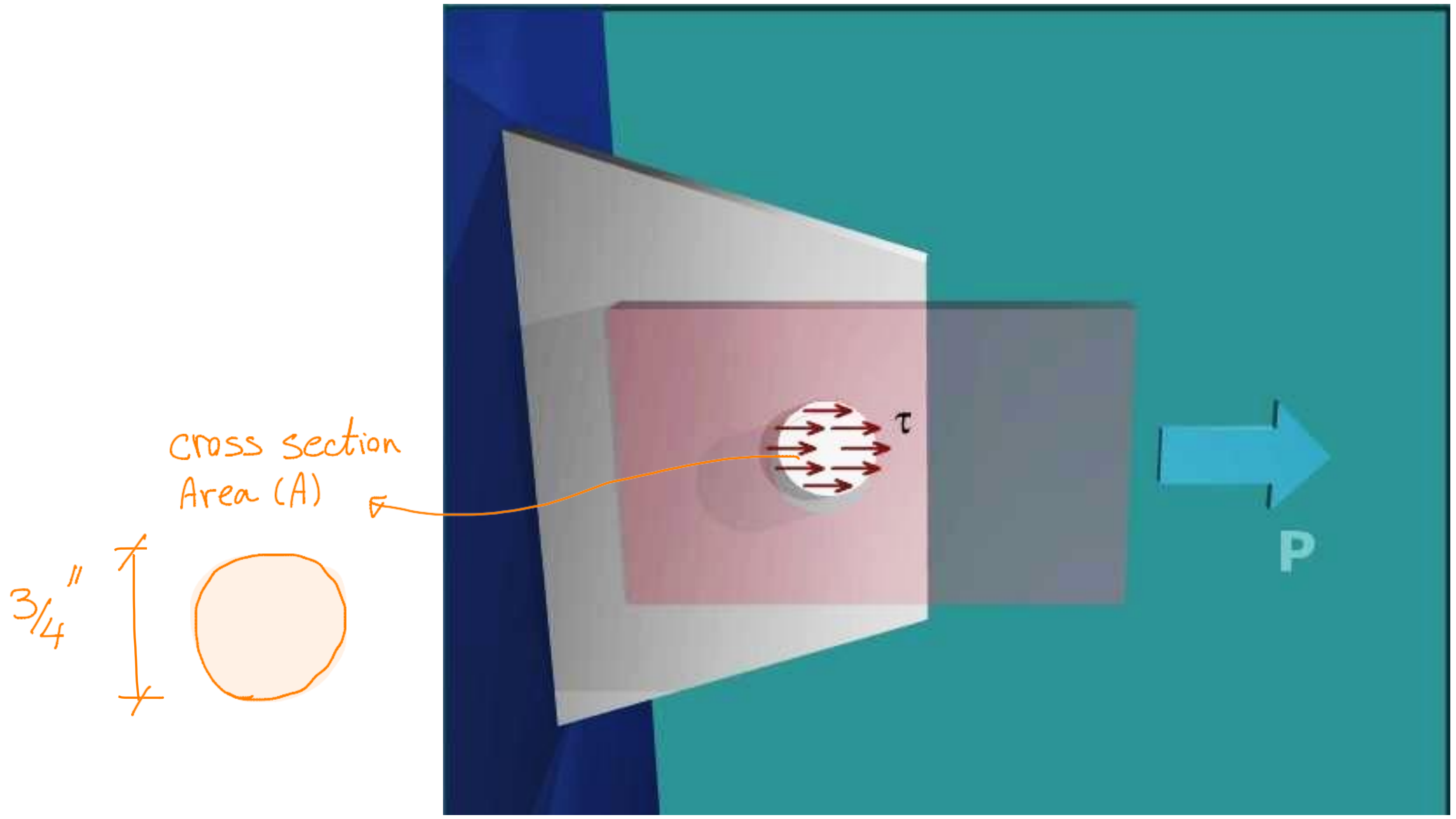
Part b) Shear Stress

Step (1) Internal force

$$F = P = 120{,}000 \ lb$$

Step (2) Cross section area

$$A = \pi \frac{d^2}{4} = \pi \frac{(3/4)^2}{4} = 0.441 \ in^2$$



Step (3) Shear stress

$$\tau = \frac{F}{A} = \frac{120{,}000 \ lb}{0.441 \ in^2} = 271{,}600 \ lb/in^2$$

$$\boxed{\tau = 271{,}600 \ psi}$$

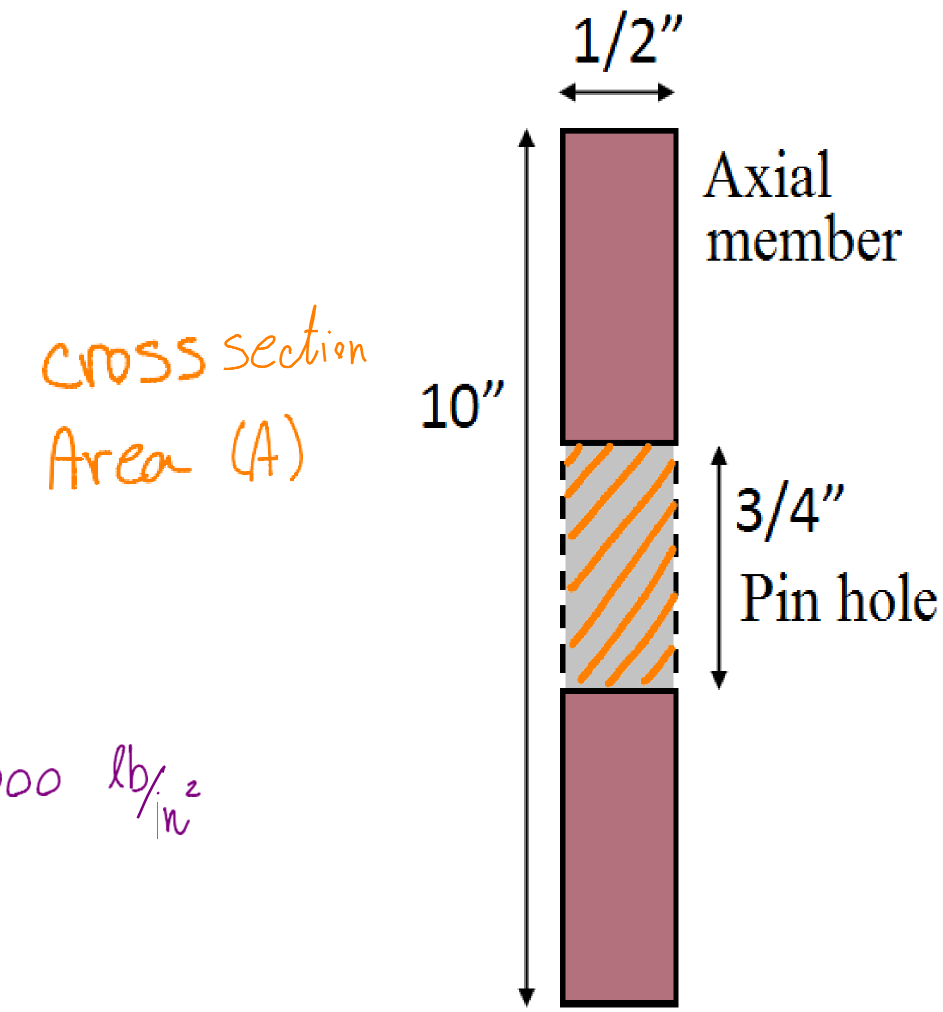
Part c) Bearing Stress

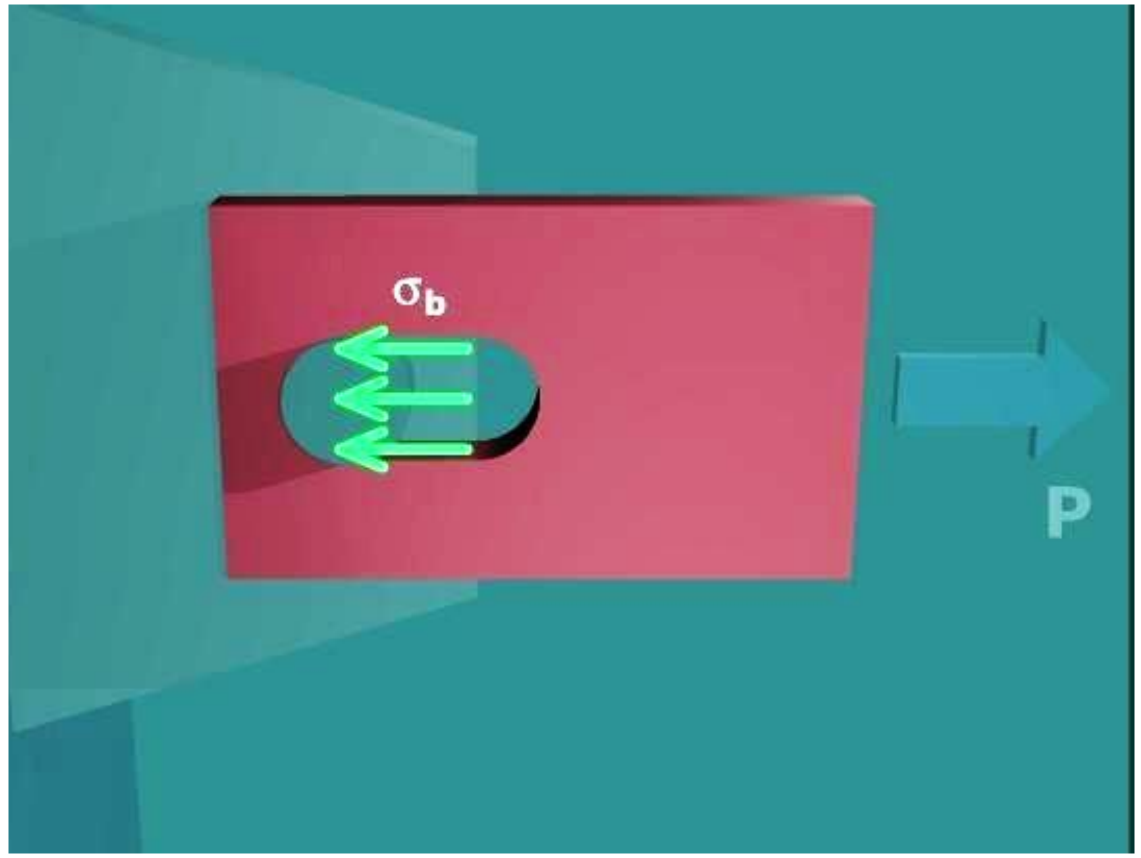
Step (1) Internal force

$$F = P = 120{,}000\ lb$$

Step (2) Cross section area

$$A = d \times t = \tfrac{1}{2}'' \times \tfrac{3}{4}'' = \tfrac{3}{8}\ in^2$$



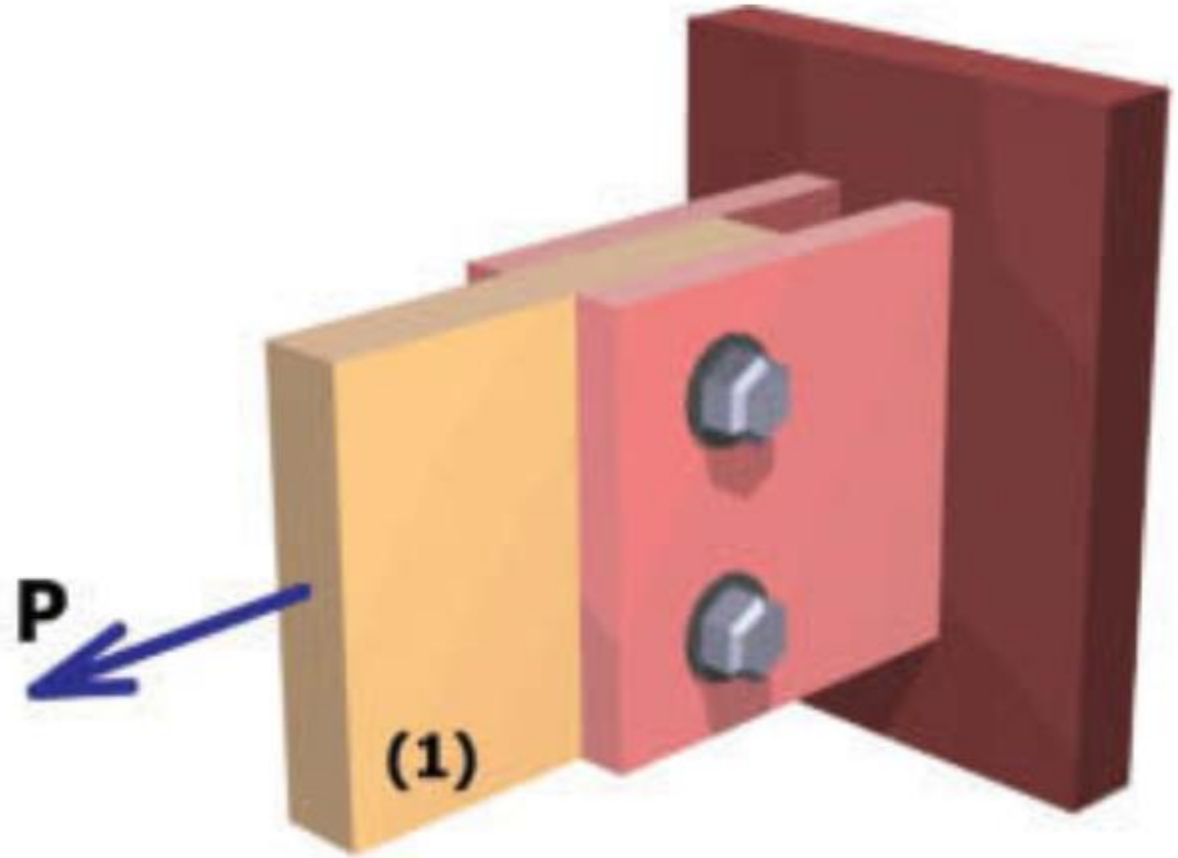


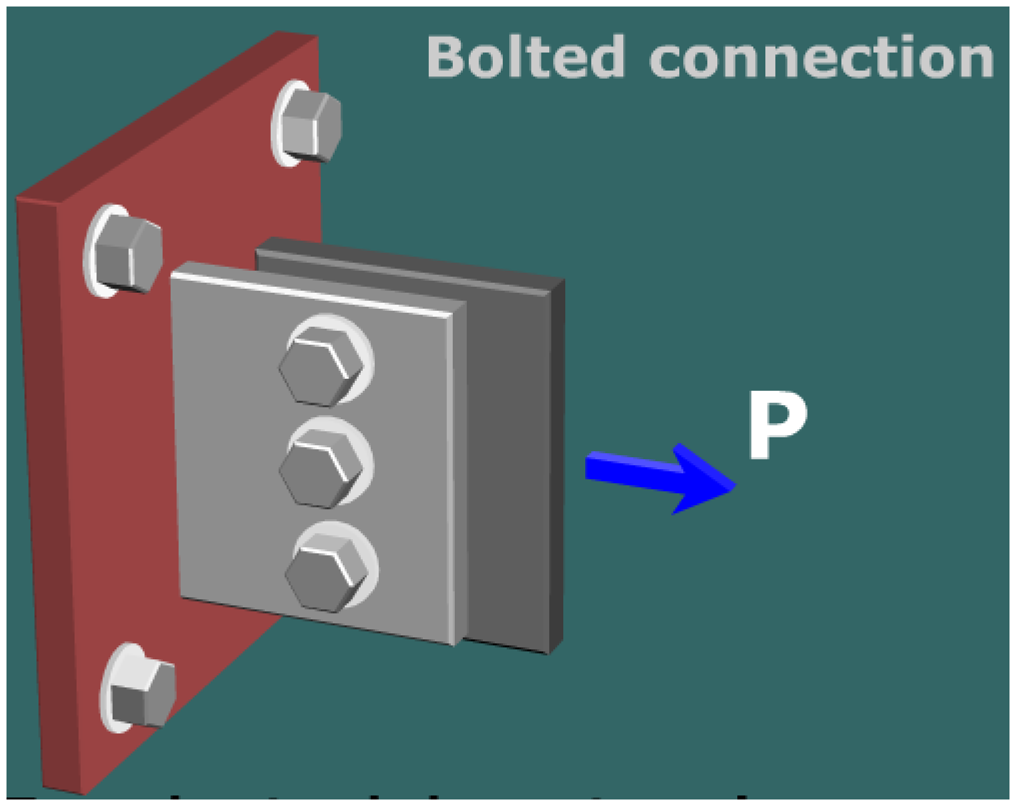
Step (3) Bearing (normal) stress

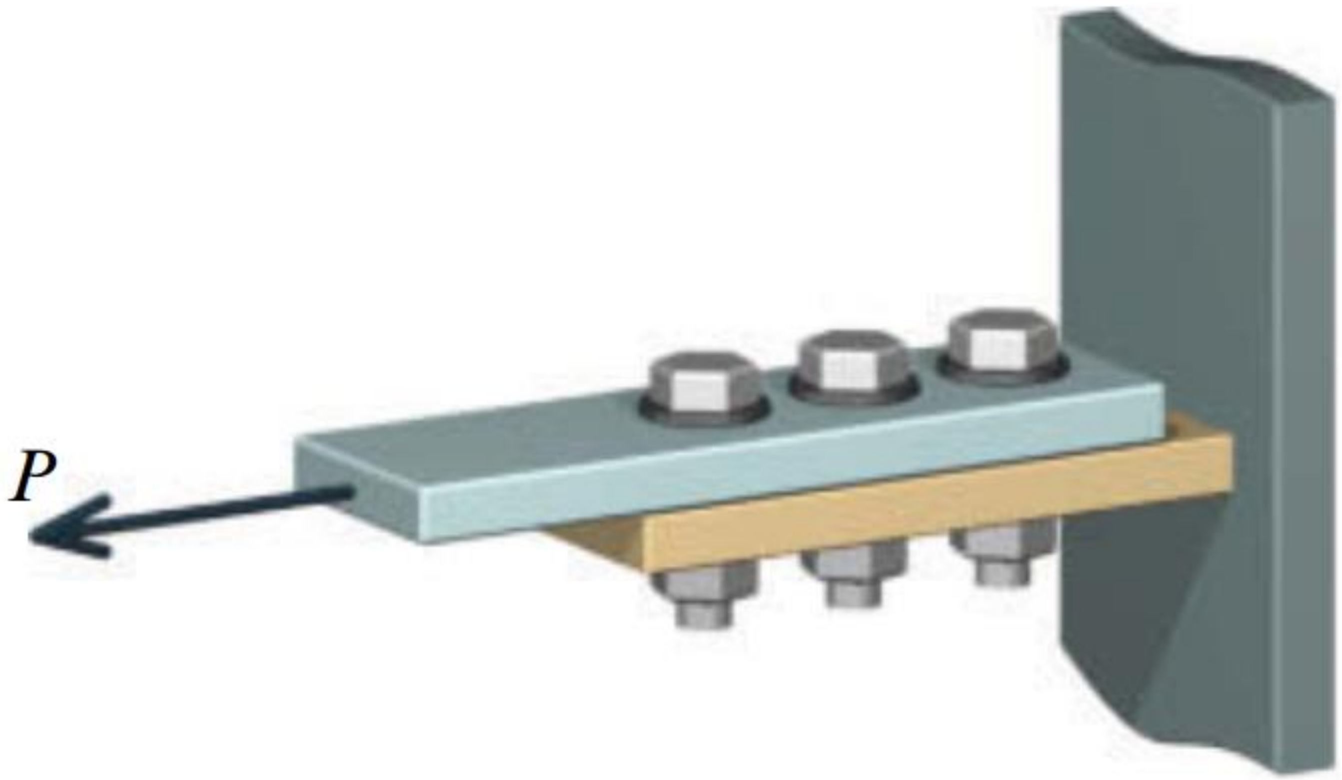
$$\sigma_b = \frac{F}{A} = \frac{120{,}000\ lb}{3/8\ in^2} = 320{,}000\ lb/in^2$$

$$\boxed{\sigma_b = 320{,}000\ psi}$$

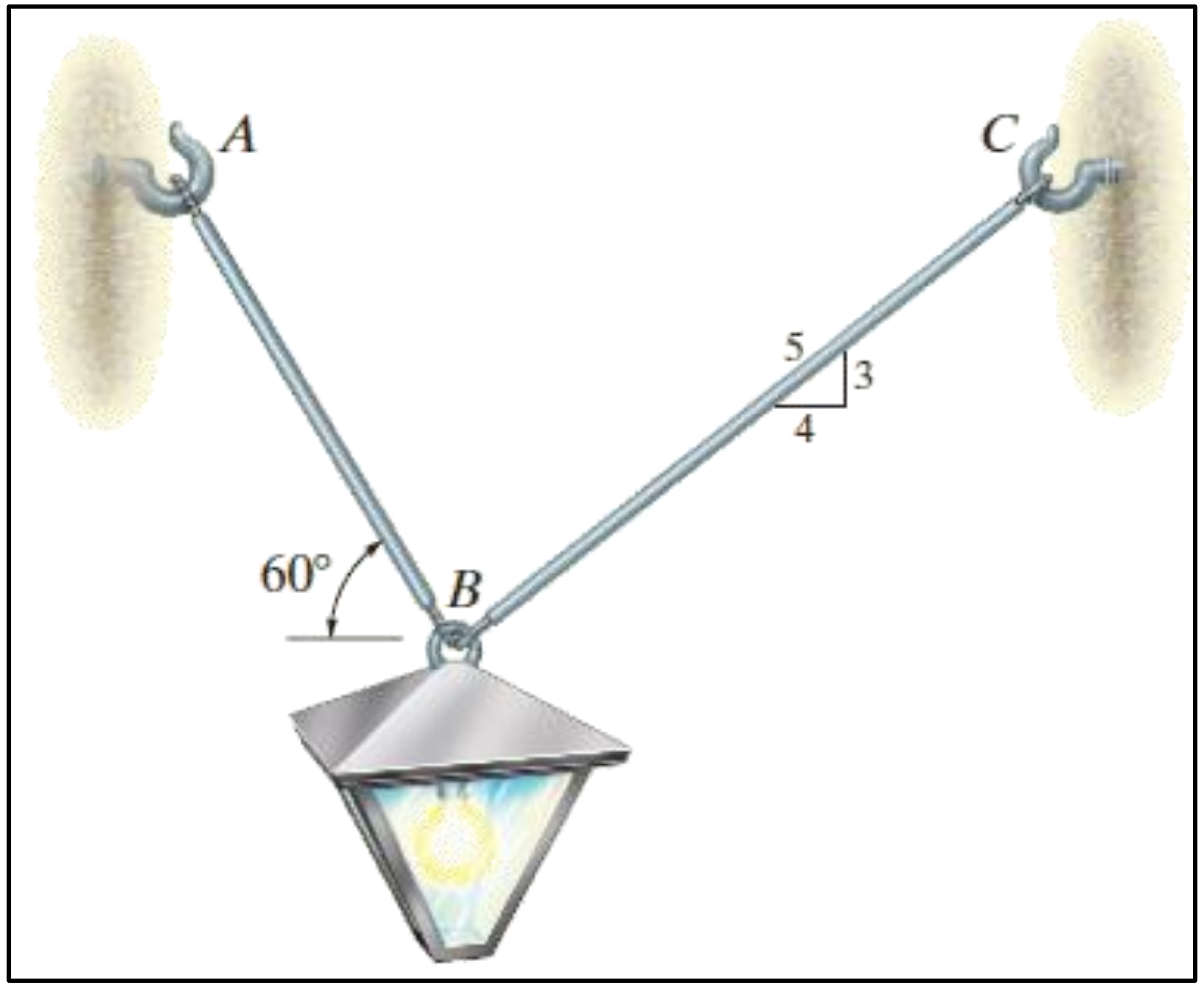
Let’s **think** together

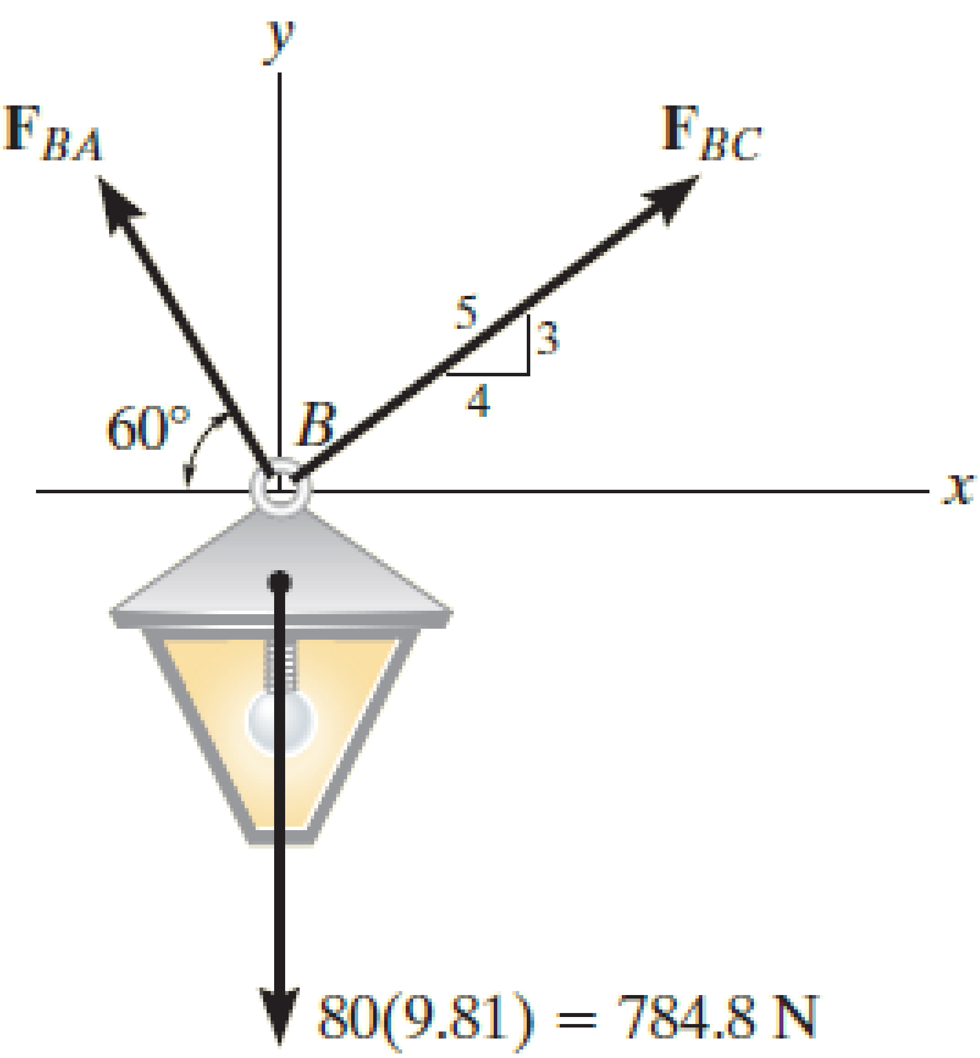






Example 3: The 80-kg lamp is supported by two rods AB and BC as shown below. If AB has a diameter of 10 mm and BC has a diameter of 8 mm, determine the average normal stress in each rod..





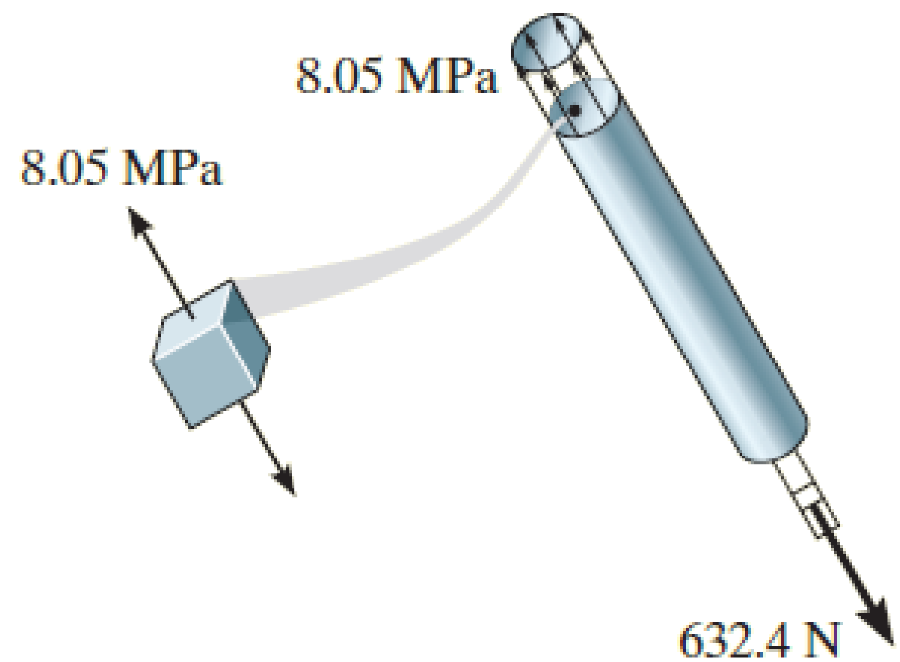
$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{BC}\left(\tfrac{4}{5}\right) - F_{BA}\cos 60^\circ = 0$$

$$+\uparrow \Sigma F_y = 0; \quad F_{BC}\left(\tfrac{3}{5}\right) + F_{BA}\sin 60^\circ - 784.8\text{ N} = 0$$

$$F_{BC} = 395.2\text{ N}, \qquad F_{BA} = 632.4\text{ N}$$

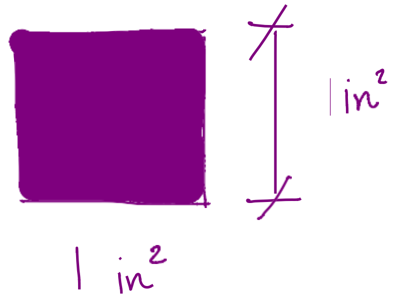
$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{395.2\text{ N}}{\pi(0.004\text{ m})^2} = 7.86\text{ MPa}$$

$$\sigma_{BA} = \frac{F_{BA}}{A_{BA}} = \frac{632.4\text{ N}}{\pi(0.005\text{ m})^2} = 8.05\text{ MPa}$$



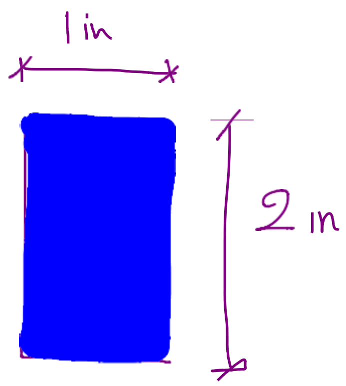
Example 4: During the tension test, the wooden specimen is subjected to a force P = 2000 lb Calculate the average normal stress . Also, find the average shear stress developed along section *a–a of* the specimen.

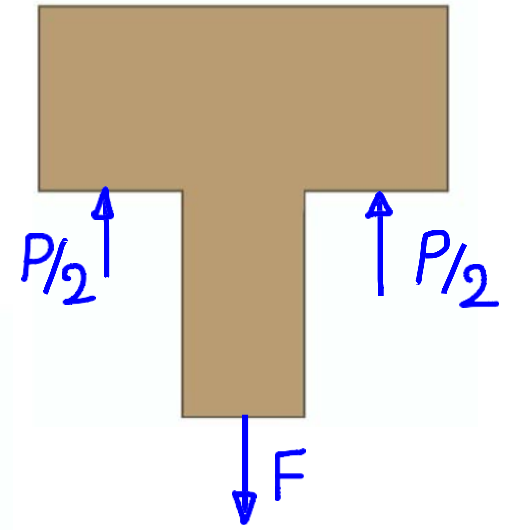
$$Normal\ stress\ \sigma = \frac{P}{A} = \frac{2000}{1*1} = 2000\ psi$$

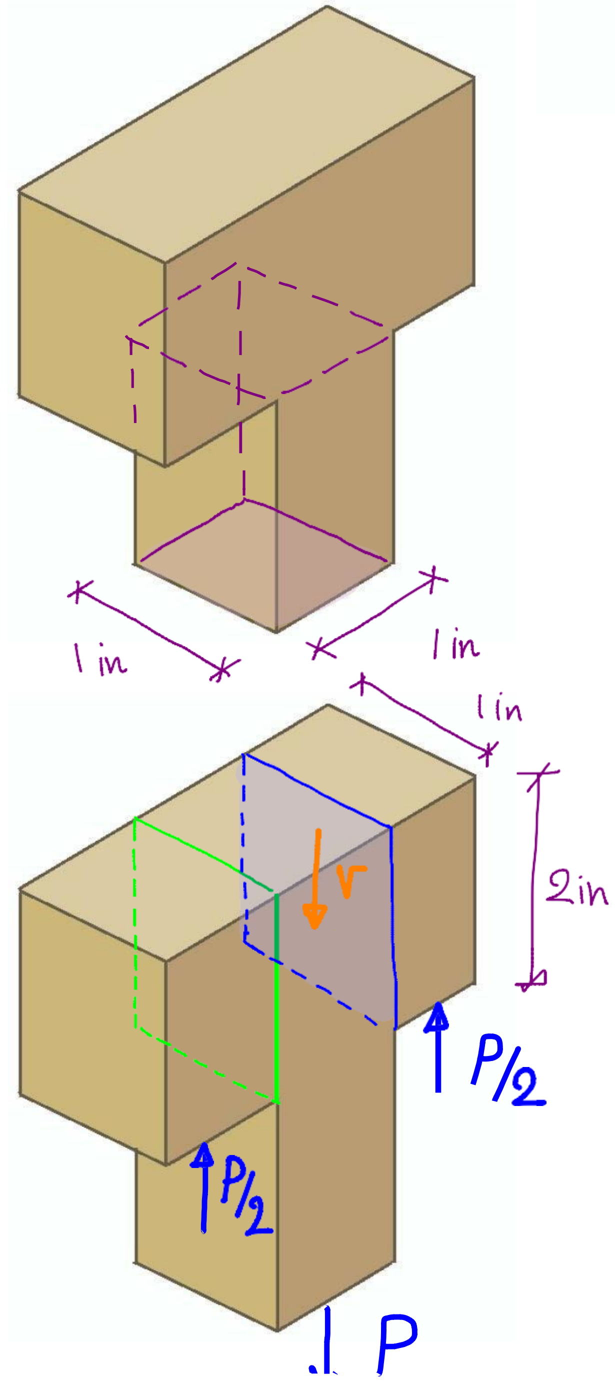


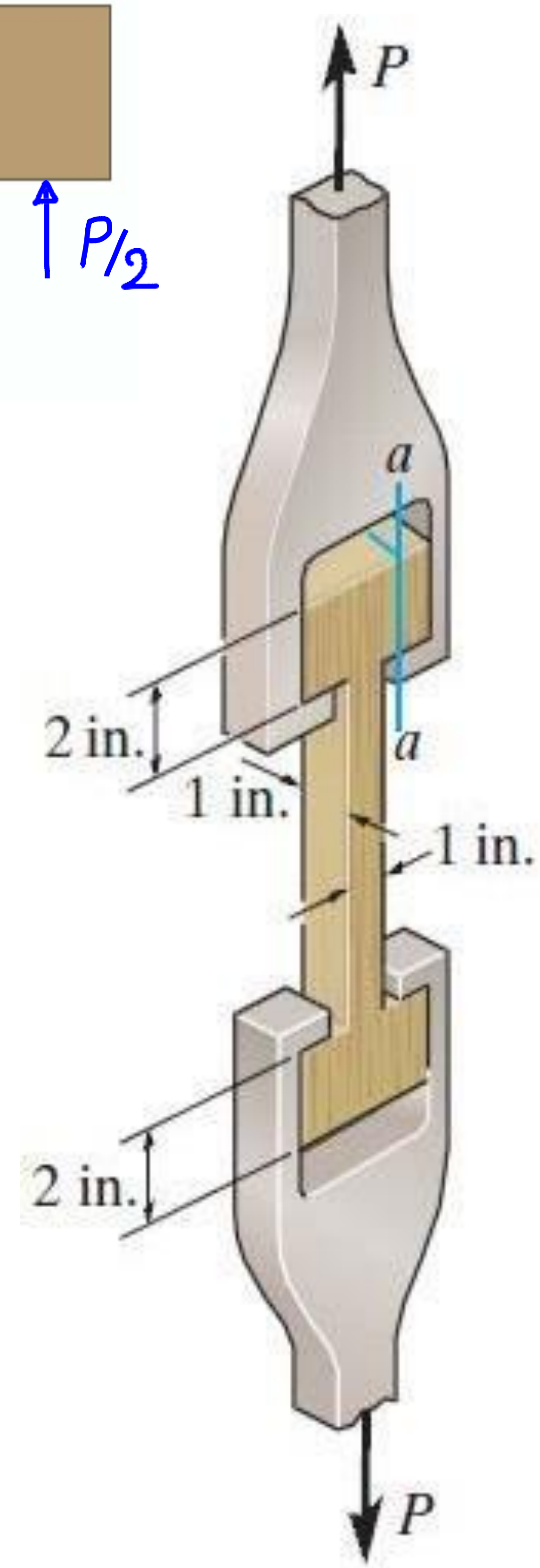
$$Shear\ stress\ \tau = \frac{P}{2A}\ or\ \frac{P/2}{A}$$

$$\frac{1000}{2*1} = 500\ psi$$

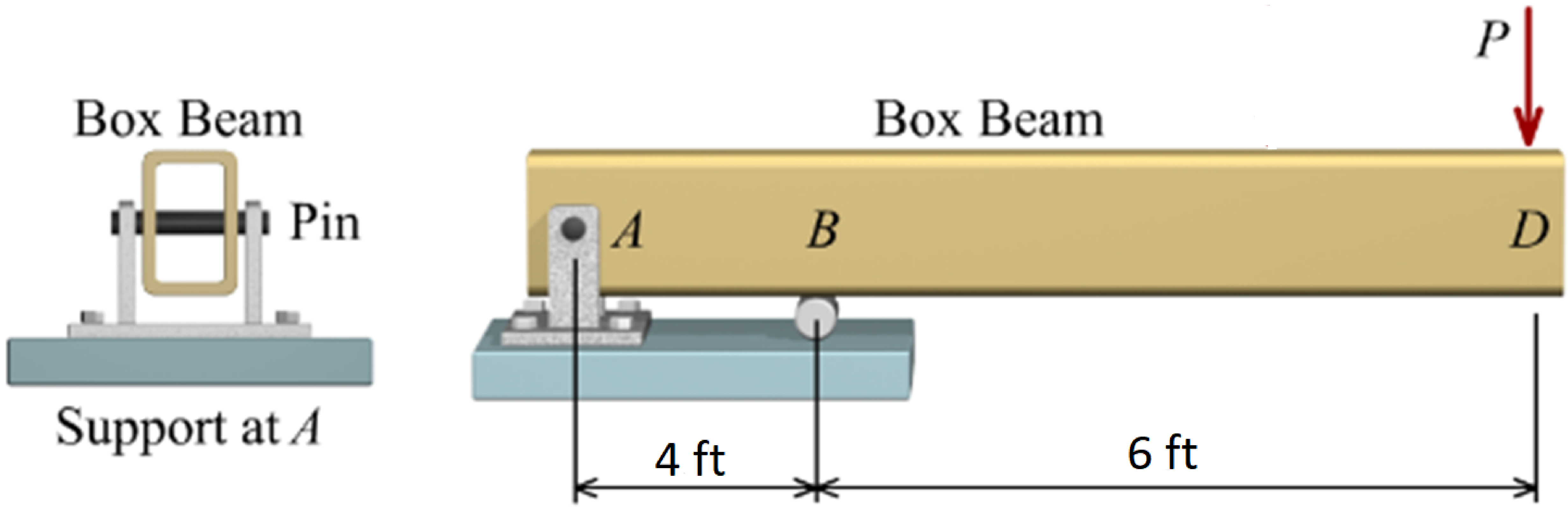








Example 5: A beam is constructed from a hollow rectangular box beam shape. The beam is supported by a pin support at A and a roller support at B. The pin at A has a diameter of 1.25 in. The concentrated load of P=3,000 lb is applied to the beam at D. Determine the shear stress in the pin at A.

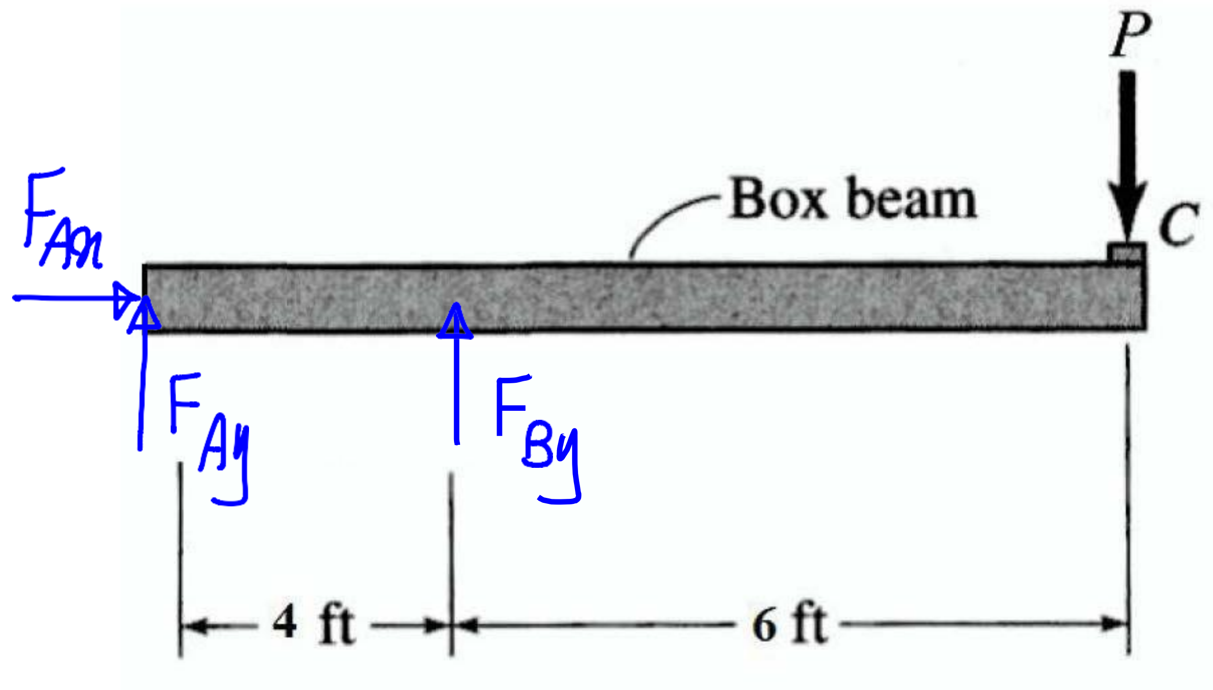


$$\Sigma F_x = 0 \Rightarrow F_{Ax} = 0$$

$$\Sigma F_y = 0 \Rightarrow F_{Ay} + F_{By} = P$$

$$\Sigma M_B = 0 \Rightarrow F_{Ay} \times 4 + P \times 6 = 0$$

$$F_{Ay} = \frac{-6}{4} P = -4500 \text{ lb}$$



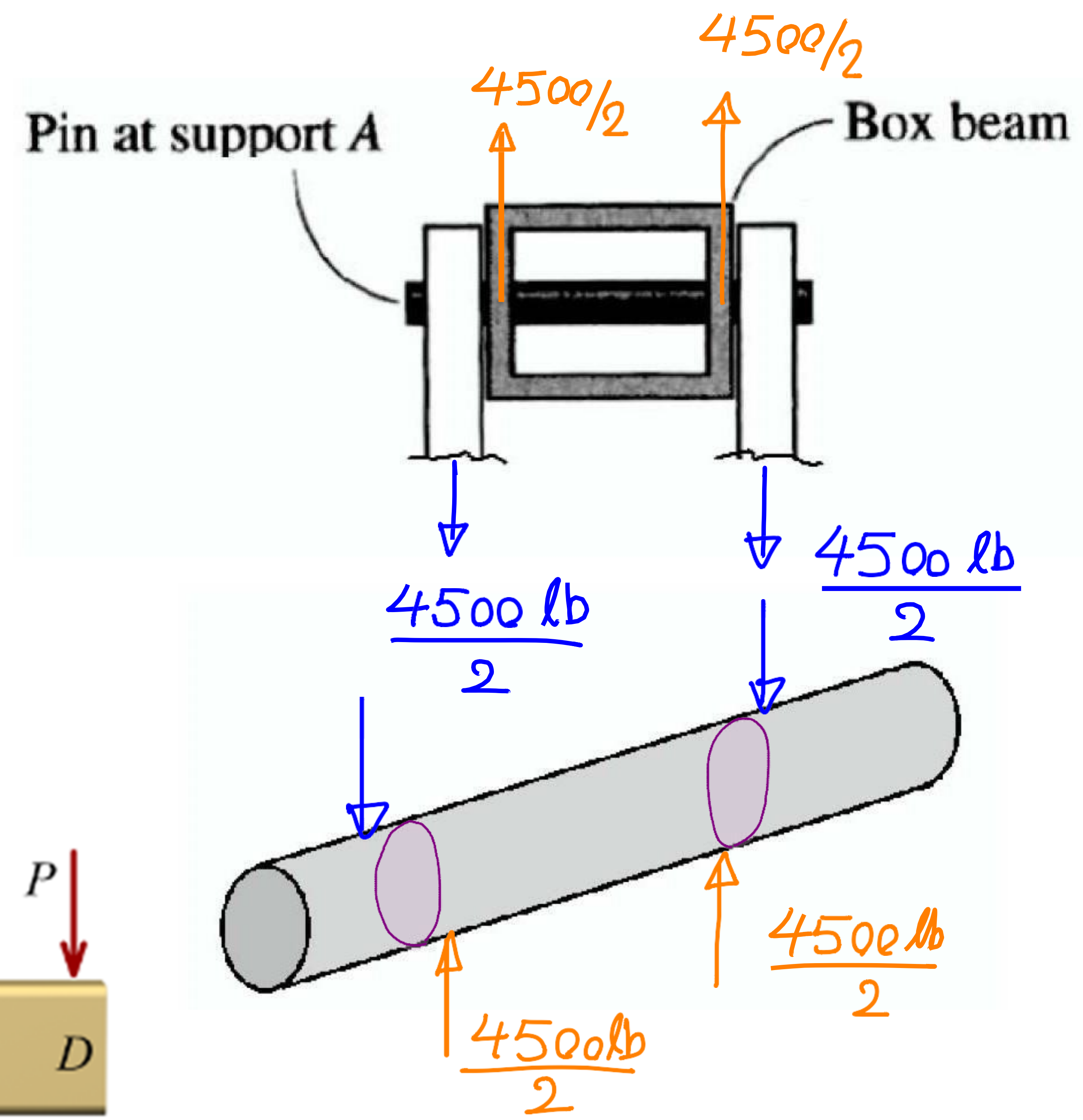
Stress developed at Pin

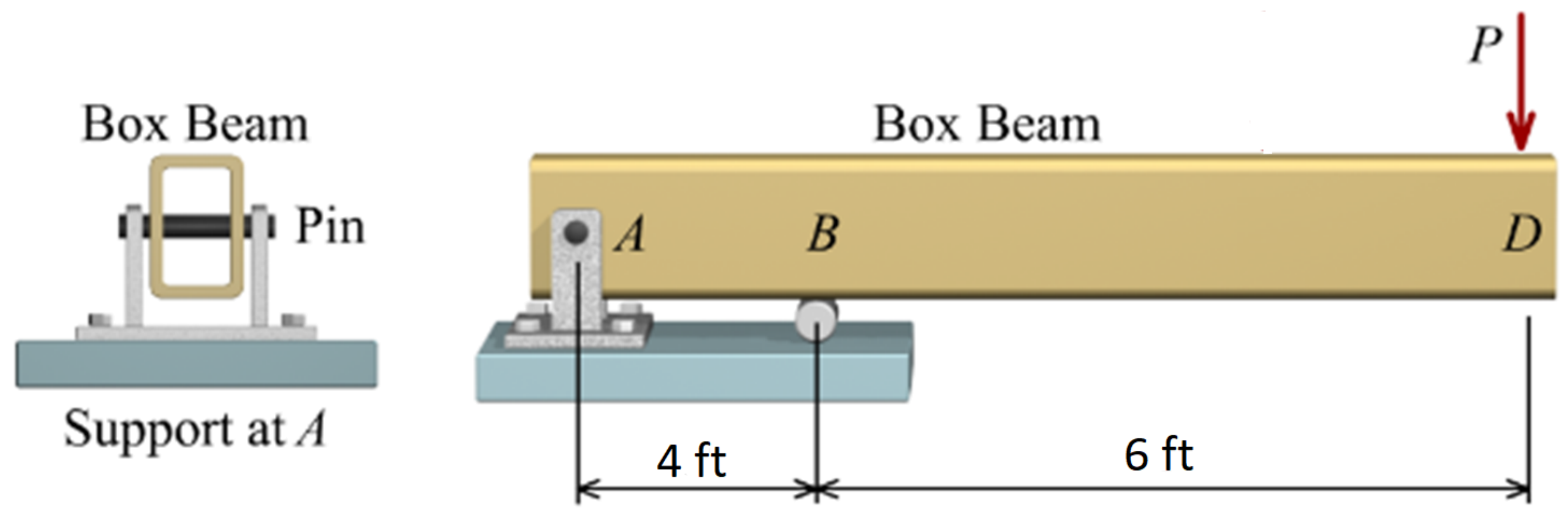
$$A = \frac{\pi d_{pin}^2}{4} = \frac{\pi \times 1.25^2}{4} = 1.227 \text{ in}^2$$

$$F = 4500/2 = 2250 \text{ lb}$$

$$\tau = \frac{F}{A} = \frac{2250 \text{ lb}}{1.227} = 1834 \text{ lb/in}^2$$

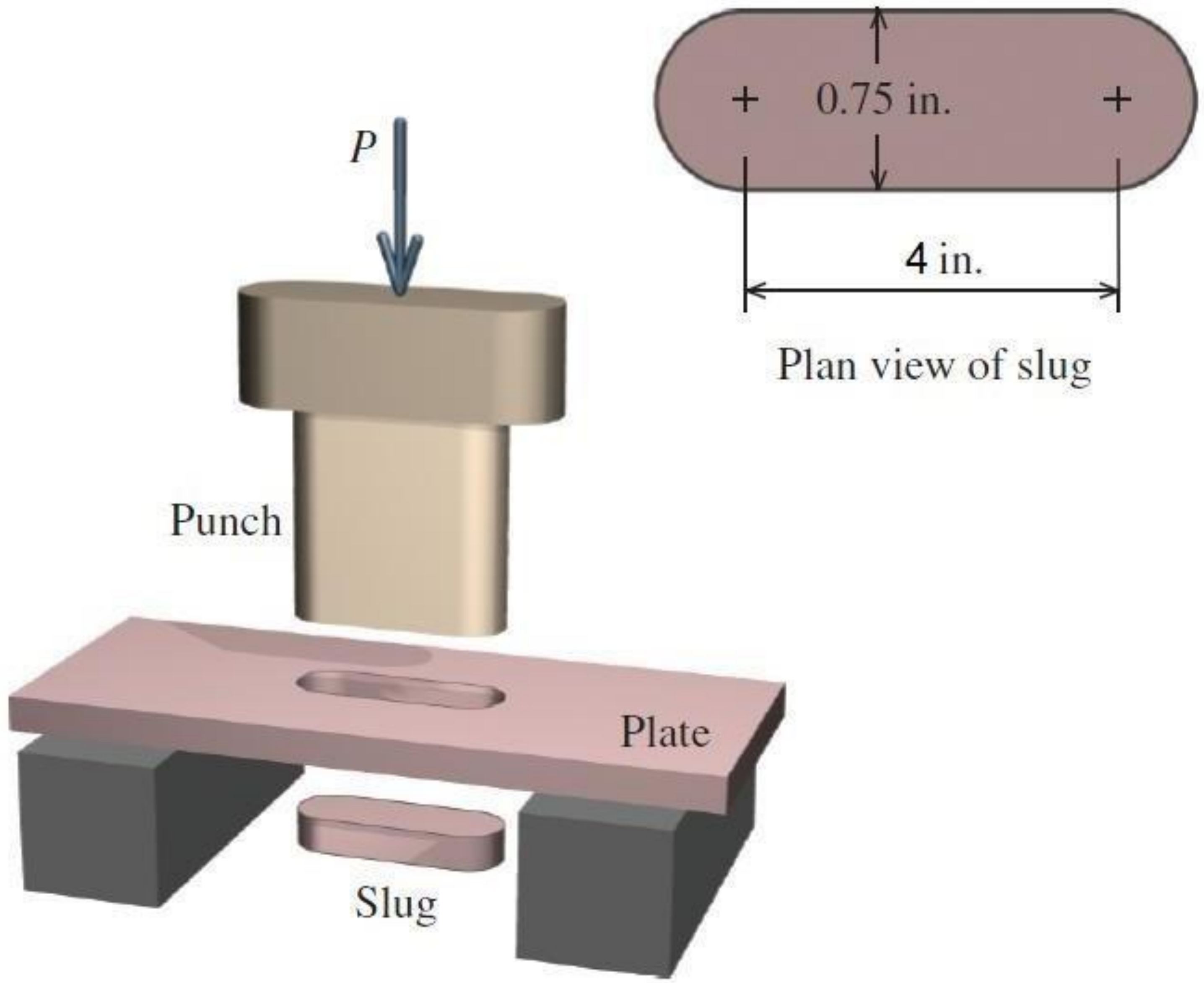
$$= 1834 \text{ psi}$$





Punching shear failure in composite wood block specimens

Example 6: A hydraulic punch press is used to punch a slot in a 0.50-in.-thick plate, as illustrated in Figure. If the plate shears at a stress of 18 ksi, determine the minimum force *P required* to punch the slot.



① Area

$$A = \left[2 \times 4 + \left(\frac{\pi d}{2}\right) \times 2\right] \times t$$

$$A = \left[2 \times 4 + \pi \times 0.75\right] \times \frac{1}{2}$$

$$A = 5.18 \text{ in}^2$$

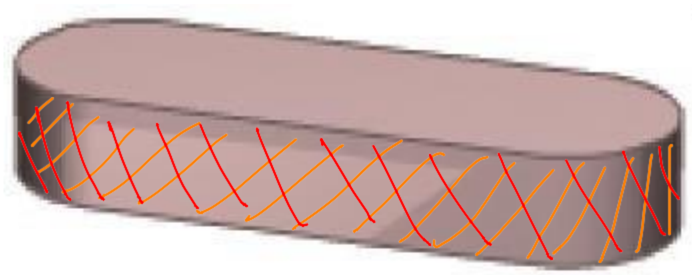
Stress

$$\tau = \frac{V}{A}$$

$$18 \text{ ksi} = \frac{V}{5.18 \text{ in}^2} \Rightarrow V = 5.18 \text{ in}^2 \times 18 \text{ kips/in}^2$$

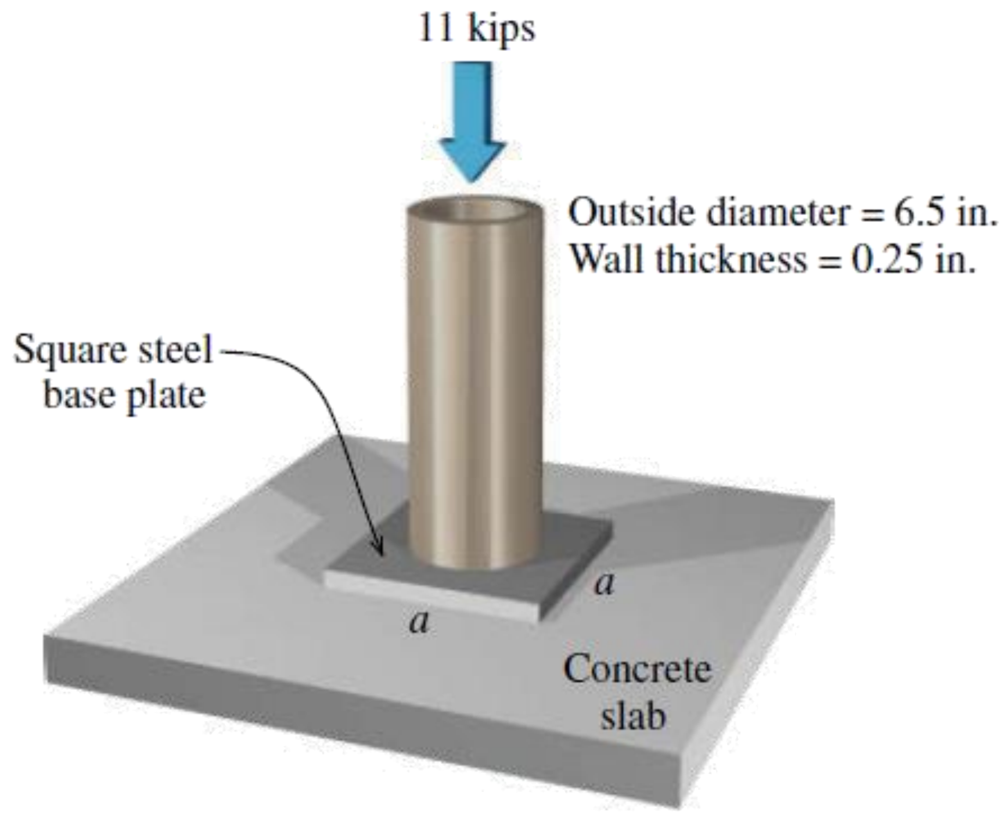
$$V = 93.24 \text{ kips}$$

$$\boxed{P = 93.24 \text{ kips}}$$



Slug

Example 7: A steel pipe column (6.5 in. outside diameter; 0.25 in. wall thickness) supports a load of 11 kips. The steel pipe rests on a square steel base plate, which in turn rests on a concrete slab.
(a) Determine the bearing stress between the steel pipe and the steel plate.
(b) If the bearing stress of the steel plate on the concrete slab must be limited to 90 psi, what is the minimum allowable plate dimension a?



$$A_{pipe} = \frac{\pi}{4}(D^2 - d^2)$$

where D = outside diameter and d = inside diameter. The inside diameter d is related to the outside diameter D by

$$d = D - 2t$$

where t = wall thickness. Therefore, with D = 6.5 in. and d = 6.0 in., the area of the pipe is

$$A_{pipe} = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}[(6.5\text{ in.})^2 - (6.0\text{ in.})^2] = 4.9087\text{ in.}^2$$

The bearing stress between the pipe and the base plate is

$$\sigma_b = \frac{F}{A_b} = \frac{11\text{ kips}}{4.9087\text{ in.}^2} = 2.24\text{ ksi}$$

(b) The minimum area required for the steel plate in order to limit the bearing stress to 90 psi is

$$\sigma_b \geq \frac{F}{A_b} \qquad \therefore A_b \geq \frac{F}{\sigma_b} = \frac{(11\text{ kips})(1{,}000\text{ lb/kip})}{90\text{ psi}} = 122.222\text{ in.}^2$$

Since the steel plate is square, its area of contact with the concrete slab is

$$A_b = a \times a \geq 122.222\text{ in.}^2$$

$$\therefore a \geq \sqrt{122.222\text{ in.}^2} = 11.06\text{ in.}$$

Design concepts (Strength criteria)

Structural designer should select appropriate materials, shape and load carrying system so that the designed structure safely carries the applied load and remains stable and usable during its service life. One important criteria that should be met is the strength requirement which is generally satisfied by keeping the stress level (σ) below the yield stress (σ_y) of material. However, there are uncertainties in evaluating laoding and strength values.

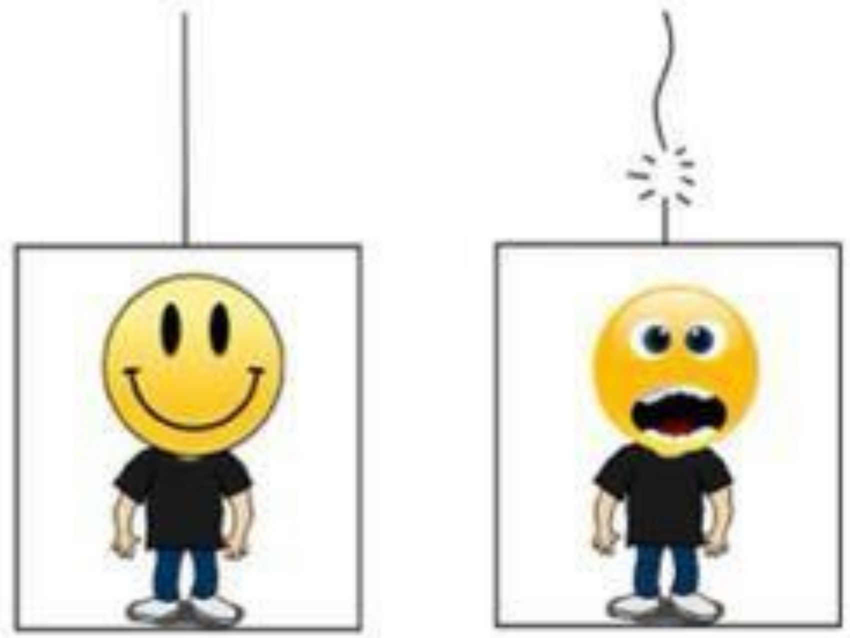
$$\sigma \leq \sigma_y$$

Uncertainties in design:

The **loads** that act on structures or machines are generally estimated, and there may be substantial variation in these loads.

The **strength** of element is not deterministic. Variation of results in different tested samples, deterioration of materials over time, internal stress and defects are some examples of what cause uncertainties in strength of elements.

To consider such uncertainties in the design, factor of safety (FS>1) is introduced.



Typical values for FS

FS is a parameter that is given by design codes. It depends on the type of loading, the material used for building the structure, importance of structure and etc. The typical FS for steel structures is 1.67 while soil design FS is about 2-3.

Determining FS in current structures

Sometimes it is needed to determine the FS in the currently built structure. Use the following equation to do so.

$$FS = \frac{\sigma_{failure}}{\sigma_{actual}} \quad \text{or} \quad FS = \frac{\tau_{failure}}{\tau_{actual}}$$

σ_{actual}: Actual stress level in the structure under loading

$\sigma_{failure}$: Failure stress capacity of materials (typically σ_y)

Note: If structure has different components with different FS, the minimum FS value is considered as the FS of structure.

Example 8 The uniform beam is supported by two rods AB and CD that have cross-sectional areas of 10 mm^2 and 15 mm^2,respectively. If the failure stress for each rod is 570 MPa, Determine the factor of safety of each road. The intensity w of the distributed load 5 kN/m.
Are they good or bad?